VAN NOSTRAND'S
NAUTICAL MANUALS

NAVIGATION AND NAUTICAL ASTRONOMY

COFFIN-COLLINS

D. VAN NOSTRAND COMPANY
NEW YORK

NAVIGATION AND NAUTICAL ASTRONOMY

H. O. No. 9 PART. II

USEFUL TABLES

Comprising Part II of the

AMERICAN PRACTICAL NAVIGATOR

By Nathaniel Bowditch

A collection of about fifty tables useful for the solution of navigational problems. This set of tables is the *only* one necessary for the use with Coffin's "Navigation and Nautical Astronomy."

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NAVIGATION

AND

NAUTICAL ASTRONOMY

PREPARED ORIGINALLY
FOR THE USE OF THE U.S. NAVAL ACADEMY

BY

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ELEVENTH EDITION, REVISED AND ENLARGED

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154123
27/1/20

NEW YORK
D. VAN NOSTRAND COMPANY
25 PARK PLACE
1919

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PUBLISHER'S NOTE

THE method of solving problems in modern navigation having changed somewhat, together with a change in the arrangement of the present Nautical Almanac, necessitates a complete revision of the text and examples in this well-known work on Navigation and Nautical Astronomy. A part of the original text and many of the examples in the previous edition have been eliminated, as they are now obsolete. Several necessary chapters have been added. The chapter on the Sumner line has been changed; a chapter dealing with all cases met with in the New Navigation has been introduced. Extracts from the Nautical Almanac for the year 1918, which have reference to the examples in this work, are in the appendix, so that with Hydrographic Office Pub. No. 9, Part II, Useful Tables, a complete work in Navigation is offered. It is hoped that this work, which contains many problems of all kinds met with by the intelligent navigator, will meet the need of the student of Navigation and also be of some assistance to the navigator.

November, 1918.

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NAVIGATION AND NAUTICAL ASTRONOMY

CHAPTER I

DEFINITIONS

1. Nautical Astronomy is that branch of science which treats of the determination of a vessel's place at sea by the aid of celestial objects — the sun, moon, planets, and stars.

2. The Celestial Sphere. — An observer looking up to the heavens at night sees the stars as if they were glittering points attached to the inner surface of a great dome or vast hollow sphere of which his eye is the center. Upon this sphere imaginary circles can be traced resembling the meridians and parallels upon the surface of the earth, and a reference to these circles enables one to describe with intelligence the apparent positions and motions of the heavenly bodies. Their directions are constantly changing, and they appear to move from east to west at such a rate as to make one complete revolution in about twenty-four hours. The sun appears to move eastward among the stars so as to make one revolution a year. The moon and the other planets have motions characteristic of the orbits which they describe; measurements with precise astronomical instruments enable one to observe other motions, divided into two classes: first, into those due to refraction, parallax, and diurnal aberration, which depend upon the observer's geographical position, and, second, those due to nutation, precession, annual aberration, and proper motion, which do not depend upon the position of the observer. From data furnished by systematic observations, proof is given that these motions occur according to well-defined laws. The position of a heavenly body can be computed for any given instant. A table giving, at equal intervals of time, the places of a body as affected by the above-mentioned second class of motions is known as an ephemeris of the body. The American Ephemeris and Nautical

Almanac furnish accurate ephemerides of the principal heavenly bodies computed for several years in advance. An observer, knowing his position on the earth, can from this data in the almanac compute the direction of a celestial body at any instant; conversely, observing with a sextant or transit the directions of celestial bodies, he can determine the time and his geographical position, and it is with this latter problem that the navigator is concerned.

3. The Zenith of an observer on the earth's surface is the point of the celestial sphere vertically overhead. The *nadir* is the point vertically beneath; the plumb-line cuts the sphere above in the zenith and below in the nadir.

4. Vertical Circles, or circles of altitude, are great circles passing through the zenith and nadir, and therefore perpendicular to the observer's horizon in every direction.

5. The Horizon is the great circle of the celestial sphere whose plane passes through the observer, perpendicular to the plumb-line, and its poles are the zenith and nadir.

The visible horizon is the true circle where sky and sea meet. It is depressed below the true horizon by an amount depending upon the observer's elevation above the water. This depression is called the *dip* of the horizon.

The sensible horizon of a place is a plane conceived to touch the surface of the earth at the observer and extended to the heavens.

The rational horizon is a plane passing through the center of the earth and parallel to the sensible horizon.

6. The Meridian is the vertical circle passing through the zenith and the north and south points of the horizon.

7. The Prime Vertical is the vertical circle passing through the zenith and the east and west points of the horizon.

8. The Altitude of a body is the distance from the horizon measured on a vertical circle passing through the body. The altitude of all bodies on the sphere are included between 0° and 90°.

9. The Zenith Distance of a body is its angular distance from the zenith, and is the complement of the altitude. Altitude + zenith distance = 90°.

10. The Azimuth, or true bearing, is the angle at the zenith between the observer's meridian and the vertical circle passing through the body. It is measured by the arc of the horizon intercepted between the north or south point and the foot of this vertical. In all cases a body will change its azimuth most

rapidly when on the meridian, and least rapidly when on the prime vertical.

11. The Amplitude is the angle between the prime vertical and that vertical circle which passes through the body when the latter is on the horizon. The amplitude is the complement of the azimuth, and is reckoned from east or west towards north or south. Thus, if the azimuth were N. 85° W., the corresponding amplitude would be W. 5° N.

12. The Celestial Equator is the great circle of the sphere whose plane is perpendicular to the earth's axis. It therefore coincides with or is parallel to the terrestrial equator.

13. The Earth's Axis produced is the axis of the celestial sphere. It cuts the sphere in the north and south poles of the equator, for brevity called the *north* and *south poles*.

14. Hour Circles are great circles of the celestial sphere passing through the north and south poles, perpendicular to the celestial equator. They correspond exactly to the meridians of the earth. The hour circle passing through the zenith coincides with the meridian.

15. The Declination of a body is its angular distance north or south of the equinoctial, and is measured by the arc of the hour-circle passing through the body. Declination ranks in the heavens with latitude on earth; thus, a star whose declination is 30° N. will pass the meridian in the zenith of all places in latitude 30° N.

16. Polar Distance is the body's angular distance from the elevated pole. When the body is on the celestial equator (declination 0°) its polar distance is 90°.

When the declination and the observer's latitude have the same name, the polar distance is the complement of the declination; but when they are of contrary names the polar distance is the sum of the declination and 90°. Thus if the observer be in north latitude, and the declination be 30° N., the polar distance would be 60°; but if the declination were 30° S., the polar distance would be 120°.

17. The Hour Angle of a body is the angle at the pole between the meridian of the observer and the hour circle passing through that body; it may also be regarded as the arc of the equinoctial intercepted between those circles. It is measured toward the west as a positive direction through the 24 hours, or 360°.

18. The Ecliptic is the great circle representing the path in which, by reason of the annual revolution of the earth, the sun appears to move in the celestial sphere; the plane of the ecliptic

is inclined to that of the equinoctial at an angle of 23° 27½′, and this inclination is called the *obliquity of the ecliptic.*

19. The Equinoxes are those points at which the ecliptic and the equinoctial intersect, and when the sun occupies either of these positions the days and nights are of equal length throughout the earth. The *Vernal Equinox* is that point at which the sun appears to an observer on the earth when passing from southern to northern declination, and the *Autumnal Equinox* that point at which it appears when passing from northern to southern declination. The vernal equinox is also designated as the *First Point of Aries*, and is used as an origin for reckoning right ascension.

The Solstices are the points in the ecliptic 90° from the equinoxes. The sun is in the summer solstice about June 21; in the winter solstice about December 21.

The Equinoctial Colure is the hour-circle passing through the equinoxes. The *Solstitial Colure* is the hour-circle passing through the solstices.

20. Right Ascension is the angle at the celestial pole between the body's hour-circle and another which passes through the vernal equinox or First Point of Aries. It is measured from the vernal equinox toward the east through 24 hours or 360°.

The Sidereal Time at any point of observation is equal to the right ascension of the observer's meridian. It also equals the hour-angle of the vernal equinox.

21. The Latitude of a place is the declination of the zenith of the place. It is also equal to the altitude of the elevated or north pole. Its complement is called the *co-latitude.*

The Longitude of a place is the arc of the equator intercepted between the meridian of the place and the prime meridian at Greenwich.

22. Coördinates. — It will be observed that the horizon and equator are of great importance to the navigator. They are the *primary circles*, and the vertical circles and hour-circles perpendicular to them are the *secondary circles.*

Reference Circles		Coördinates	
Primary	Secondary	Primary	Secondary
Horizon	Vertical circle	Azimuth	Altitude
Equator	Hour circle	Right Ascension	Declination
Equator	Hour circle	Hour angle	Declination

The Greenwich Date is the astronomical time at Greenwich at the instant the observation is taken.

Some of the preceding definitions are illustrated in Fig. 1.

23. Navigation is the science of conducting a vessel from place to place and determining by means of dead reckoning, or observation of the heavenly bodies, the position of a vessel at sea.

A Nautical Mile is equal to the mean length of a minute of latitude and is reckoned as 6080 feet. As it is impracticable to construct speed recording instruments, such as patent logs, to register the length of a nautical mile, since it varies in different latitudes, it is necessary to decide upon a length for the nautical mile which these instruments may be constructed to record. The length chosen is 6080 feet. **The knot** — the unit of speed in navigation — is the speed of one nautical mile an hour. A vessel steaming 14 nautical miles an hour is said to be steaming 14 knots, and this should never be expressed as "14 knots per hour."

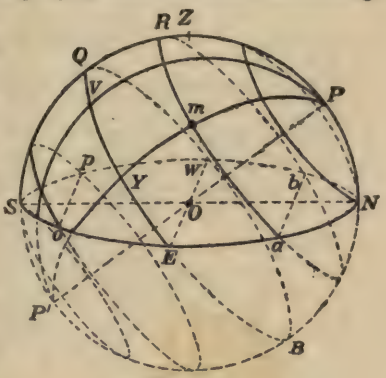


FIGURE 1

O, place of observer; *Z* = Zenith.
SENW, Horizon.
POP', line parallel to the Axis of the Earth.
P and *P'*, Poles of the Heavens.
EQWB, Celestial Equator, or Equinoctial.
V, Vernal Equinox, or "First point of Aries."
PVP', Equinoctial Colure, or Zero Hour-circle.
m, Star.
Ym, Star's Declination; *Pm*, North-polar Distance.
Angle *mPR* = *QY*, Star's (eastern) Hour-angle = 24h minus Star's (western) Hour angle.
Angle *VPm* = arc-*VY*, Star's Right Ascension. Sidereal time at the moment = 24h minus angle *VPQ*.

The Course steered is the angle between a meridian and the keel or the fore and aft line of the vessel. The course made good is the angle between the meridian and the rhumb-line joining the place of departure and the place arrived.

Variation is the angle between the true and magnetic meridian at the place, or the angle between the directions of the true and magnetic north. Variation is named East or West according as the magnetic North lies East or West of the meridian of the place.

Deviation is the angle between the magnetic and compass meridian, or the angle between the direction of magnetic and

compass north, and is named East or West according as the North-seeking end of the compass needle lies to the east or west of the magnetic meridian.

Leeway is measured by the estimated angle between a vessel's keel and her wake.

CHAPTER II

THE NAVIGATOR'S COMPASS.—INSTRUMENTS

24. The Compass is an instrument consisting of a group of magnets (shown in Fig. 2) attached to a graduated circle pivoted at the center and allowed to swing freely in a hori-

FIGURE 2

zontal plane. It has a tendency, when not affected by disturbing magnetic features within the vessel, to lie with its magnetic axis in the plane of the earth's magnetic meridian, thus affording a means of determining the azimuth, from that

meridian, of the vessel's course and of all visible objects, terrestrial or celestial.

The circular card of the compass is divided into 360°, frequently numbered from 0° at North, and South to 90° at East, and West; also into thirty-two divisions of $11\frac{1}{4}°$ each, called *points*, the latter being further divided into *half-points* and *quarter-points*. A system of numbering the degrees from 0° to 360°, always increasing toward the right, is shown in Fig. 2.

The names of the whole points, together with fractional points, are given in the following table, which shows also the degrees, minutes, and seconds from North or South to which each division corresponds:

	Points	Angular measure				Points	Angular measure		
NORTH TO EAST					EAST TO SOUTH				
North:		°	′	″	East	8	90	00	05
N. ¼ E.	¼	2	48	45	E. ¼ S.	8¼	92	48	40
N. ½ E.	½	5	37	30	E. ½ S.	8½	95	37	35
N. ¾ E.	¾	8	26	15	E. ¾ S.	8¾	98	26	10
N. by E.	1	11	15	00	E. by S.	9	101	15	05
N. by E. ¼ E.	1¼	14	03	45	ESE. ¾ E.	9¼	104	03	40
N. by E. ½ E.	1½	16	52	30	ESE. ½ E.	9½	106	52	35
N. by E. ¾ E.	1¾	19	41	15	ESE. ¼ E.	9¾	109	41	10
NNE.	2	22	30	00	ESE.	10	112	30	05
NNE. ¼ E.	2¼	25	18	45	SE. by E. ¾ E.	10¼	115	18	40
NNE. ½ E.	2½	28	07	30	SE. by E. ½ E.	10½	118	07	35
NNE. ¾ E.	2¾	30	56	15	SE. by E. ¼ E.	10¾	120	56	1
NE. by N.	3	33	45	00	SE. by E.	11	123	45	00
NE. ¾ N.	3¼	36	33	45	SE. ¾ E.	11¼	126	33	45
NE. ½ N.	3½	39	22	30	SE. ½ E.	11½	129	22	30
NE. ¼ N.	3¾	42	11	15	SE. ¼ E.	11¾	132	11	15
NE.	4	45	00	00	SE.	12	135	00	00
NE. ¼ E.	4¼	47	48	45	SE. ¼ S.	12¼	137	48	45
NE. ½ E.	4½	50	37	30	SE. ½ S.	12½	140	37	30
NE. ¾ E.	4¾	53	26	15	SE. ¾ S.	12¾	143	26	15
NE. by E.	5	56	15	00	SE. by S.	13	146	15	00
NE. by E. ¼ E.	5¼	59	03	45	SSE. ¾ E.	13¼	149	03	45
NE. by E. ½ E.	5½	61	52	30	SSE. ½ E.	13½	151	52	30
NE. by E. ¾ E.	5¾	64	41	15	SSE. ¼ E.	13¾	154	41	15
ENE.	6	67	30	00	SSE.	14	157	30	00
ENE. ¼ E.	6¼	70	18	45	S. by E. ¾ E.	14¼	160	18	45
ENE. ½ E.	6½	73	07	30	S. by E. ½ E.	14½	163	07	30
ENE. ¾ E.	6¾	75	56	15	S. by E. ¼ E.	14¾	165	56	15
E. by N.	7	78	45	00	S. by E.	15	168	45	00
E. ¼ N.	7¼	81	33	45	S. ¾ E.	15¼	171	33	45
E. ½ N.	7½	84	22	30	S. ½ E.	15½	174	22	30
E. ¼ N.	7¾	87	11	15	S. ¼ E.	15¾	177	11	15

SOUTH TO WEST	Points	Angular measure	WEST TO NORTH	Points	Angular measure
South	16	180 00 00	West	24	270 00 00
S. ¼ W	16¼	182 48 45	W. ¼ N	24¼	272 48 45
S. ½ W	16½	185 37 30	W. ½ N	24½	275 37 30
S. ¾ W	16¾	188 26 15	W. ¾ N	24¾	278 26 15
S. by W	17	191 15 00	W. by N	25	281 15 00
S. by W. ¼ W	17¼	194 03 45	WNW. ¾ W	25¼	284 03 45
S. by W. ½ W	17½	196 52 30	WNW. ½ W	25½	286 52 30
S. by W. ¾ W	17¾	199 41 15	WNW. ¼ W	25¾	289 41 15
SSW	18	202 30 00	WNW	26	292 30 00
SSW. ¼ W	18¼	205 18 45	NW. by W. ¾ W	26¼	295 18 45
SSW. ½ W	18½	208 07 30	NW. by W. ½ W	26½	298 07 30
SSW. ¾ W	18¾	210 56 15	NW. by W. ¼ W	26¾	300 56 15
SW. by S	19	213 45 00	NW. by W	27	303 45 00
SW. ¾ S	19¼	216 33 45	NW. ¾ W	27¼	306 33 45
SW. ½ S	19½	219 22 30	NW. ½ W	27½	309 22 30
SW. ¼ S	19¾	222 11 15	NW. ¼ W	27¾	311 11 15
SW	20	225 00 00	NW	28	315 00 00
SW. ¼ W	20¼	227 48 45	NW. ¼ N	28¼	317 48 45
SW. ½ W	20½	230 37 30	NW. ½ N	28½	320 37 30
SW. ¾ W	20¾	233 26 15	NW. ¾ N	28¾	323 26 15
SW. by W	21	236 15 00	NW. by N	29	326 15 00
SW. by W. ¼ W	21¼	239 03 45	NNW. ¾ W	29¼	329 03 45
SW. by W. ½ W	21½	241 52 30	NNW. ½ W	29½	331 52 30
SW. by W. ¾ W	21¾	244 41 15	NNW. ¼ W	29¾	334 41 15
WSW	22	247 30 00	NNW	30	337 30 00
WSW. ¼ W	22¼	250 18 45	N. by W. ¾ W	30¼	340 18 45
WSW. ½ W	22½	253 07 30	N. by W. ½ W	30½	343 07 30
WSW. ¾ W	22¾	255 56 15	N. by W. ¼ W	30¾	345 56 15
W. by S	23	258 45 00	N. by W	31	348 45 00
W. ¾ S	23¼	261 33 45	N. ½ W	31¼	351 33 45
W. ½ S	23½	264 22 30	N. ¼ W	31½	354 22 30
W. ¼ S	23¾	267 11 15	North	31¾	357 11 15
				32	360 00 00

25. The Sextant. — The *sextant* is an instrument for measuring the angle between two objects by bringing into coincidence at the eye of the observer rays of light received directly from the one and by reflection from the other, the measure being afforded by the inclination of the reflecting surfaces. While the sextant is not capable of the same degree of accuracy as fixed instruments, its measurements are exact enough for navigation.

26. The Chronometer. — The *chronometer* is a correct time measurer, differing from an ordinary clock in having the force of its mainspring rendered uniform by means of a variable lever. It is the instrument that gives us the Greenwich mean time at any instant.

27. The Artificial Horizon. — The *artificial horizon* is a small, rectangular, shallow basin of mercury, having a roof consisting of two plates of glass at right angles to each other to protect the mercury from agitation by the wind. The mercury affords a perfectly horizontal surface, which is at the same time an excellent mirror. The artificial horizon is used on shore for observing altitudes of heavenly bodies.

28. The Patent Log. — This is a mechanical contrivance for registering the distance run by a vessel through the water. It consists of a registering device, a rotator, and a log line; the rotator is a small spindle with a number of blades extending radially so as to form a spiral, and, when drawn through the water, rotates after the manner of a screw propeller; the rotator is towed from the vessel by means of a log line from 30 to 100 fathoms in length. The registering device is constructed so as to show upon a dial the distance run. The register is carried at some convenient point on the vessel's quarter.

29. The Lead. — The lead line consists of a marked line, having a lead attached to one of its ends for ascertaining the depth of water. It is a valuable aid to the navigator in shallow water particularly in thick or foggy weather, and is also of service when the vessel is out of sight of land.

Two lead lines are used for soundings — the *hand-lead*, weighing from 7 to 14 pounds, with a line marked to about 25 fathoms, and the *deep-sea lead*, weighing from 30 to 100 pounds, the line being 100 fathoms or upward in length.

30. The Sounding Machine. — This machine possesses great advantages over the deep-sea lead, in that soundings may be obtained at great depths and with rapidity and accuracy without stopping the vessel.

31. The Barometer. — The *barometer* is an instrument for measuring the pressure of the atmosphere, and is of great service to the navigator in affording a knowledge of existing meteorological conditions and of the probable changes therein. There are two kinds of barometers used — the *mercurial* and the *aneroid*.

32. The Thermometer. — The *thermometer* is an instrument for indicating temperature. The Fahrenheit thermometer is generally used on board vessels. Two thermometers, a wet and dry bulb, called a psychrometer, for determining the relative humidity of the atmosphere, are also used.

CHAPTER III

VARIATION OF COMPASS

VARIATION.—DEVIATION.—LEEWAY.—CORRECTING COURSE

33. The true course made good by a vessel is the angle between the track and the meridian passing through the vessel. Corrections known as Variation, Deviation, and Leeway are applied to the compass to obtain this true course. The three corrections combined are called the total error of the compass.

34. Variation. — Outside of any magnetic influence of the vessel (called the deviation) the north point of the freely suspended compass needle, in most all parts of the world, except on the agonic line of No Variation, is drawn either to the eastward or westward of true north. The amount it is in error of true north is known as the Variation and is called easterly when the north end of the compass needle is drawn to the eastward of true north and westerly when the needle is drawn to the westward.

The Variation chart of the world * shows the amount of variation in degrees, whether easterly or westerly, also the annual rate of change, in any locality of the world.

Given the compass course, to find the magnetic (or given the magnetic to find the true):

Imagine yourself standing at the center of the compass card and looking in the given direction; apply easterly deviation (or variation) to the right, and apply westerly deviation (or variation) to the left.

Given the true course, to find the magnetic (or given the magnetic to find the compass):

Reverse the rule; apply easterly variation (or deviation) to the left, and apply westerly variation (or deviation) to the right.

As many compass cards are now graduated from 0° to 360°, it is convenient when using them to name easterly deviations and variations +, and westerly deviations and variations −, and apply according to algebraic signs.

* H. O. Chart No. 2406.

The navigator need remember only the one word "T R E" to cover all cases in course correcting.

From a compass course to find a true, variation easterly, apply to right, "True Right East."

From a true course to find a compass course, since it is the reverse process, mentally change R to "Left."

The direction towards which the north point of the compass points, unaffected by deviation, is called magnetic north, and the course the magnetic course.

EXAMPLE 1: Given the true course N. 45° E. Var. 11° E., find the magnetic course.

True course	N. 45° E.
Var.	11° E. (left)
Mag. course	N. 34° E.

EXAMPLE 2: Given the magnetic course N. 60° W., Var. 25° W., find the true course.

Mag. course	N. 60° W.
Var.	25° W. (left)
True course	N. 85° W.

It is extremely important to be able with facility to change compass courses to magnetic or true courses and the reverse.

The above rules apply both for variation and deviation.

35. Deviation is the effect of the magnetic attraction, on the compass, of the iron used in the construction of the vessel. This attraction shows itself as the vessel swings on the different headings, by drawing the north point of the compass needle eastward or westward of the direction which it would otherwise point, which direction is the magnetic north. Deviation is, then, the angle between the magnetic meridian and the north and south line shown by the compass card disturbed by the iron in the vessel.

Variation is not disturbed by the heading of the vessel, but depends upon the vessel's position on the ocean, but deviation changes in amount on the different headings of the vessel. When the amount of deviation on certain headings is large, magnets are placed near the compass to neutralize the force causing the disturbance.

Since those causes which produce the deviation change with the geographical position, also with the kind of a cargo, the deviation table requires constant correction, and every available means must be at all times employed to keep it correct.

EXAMPLE 1: Find the magnetic course, compass course S. 79° W., Dev. 10° W.

Compass course	S. 79° W.
Dev.	10° W. (left)
Mag. course	S. 69° W.

EXAMPLE 2: Find the compass course, magnetic course N. 45° E., Dev. 17° E.

Mag. course	N. 45° E.
Dev.	17° E. (left)
Compass course	N. 28° E.

EXAMPLE 3: Find the true course, compass course N. 28° E., Dev. 17° E., Var. 5° W.

Compass course	N. 28° E.
Dev.	17° E. (right)
Mag. course	N. 45° E.
Var.	5° W. (left)
True course	N. 40° E.

When a vessel is new and the worst errors of the compass adjusted by the distribution of permanent magnets, it then becomes necessary to determine a deviation table to find the deviation of the compass on the different headings. This is found by reciprocal bearings; that is, an observer is sent ashore with a compass, at a position free from local magnetism, where his compass can be seen by the navigator of the vessel, and by means of signals each takes a bearing of the other. The difference of bearing of the standard reversed and the shore bearing is the deviation on that heading.

EXAMPLE 1:

Time	Vessel's heading by standard compass	Bearing of the shore compass from vessel's standard compass	Bearing of vessel's compass from the shore compass	Deviation of standard compass
h m s	▫	° ′	° ′	° ′
8 30 05	▫	216 00	30 10	− 5 50
8 34 20	15	206 30	25 15	− 1 15
8 39 30	30	201 20	22 45	+ 1 25
8 44 00	45	191 50	20 20	+ 8 30
8 50 30	60	187 40	17 00	+ 9 20
8 55 00	75	185 45	16 50	+ 11 05
9 00 00	90	184 10	15 45	+ 11 35
	and so on the same way around the compass			

This deviation table may be also obtained by "swinging ship," that is the vessel can put out to sea, away from all land, in mid ocean, and by being steadied on each heading or point of the compass, then at a given instant of time have the bearing of the sun observed with an azimuth circle (an instrument fitted to the compass, at one end of which there is a mirror reflecting the image of the sun, and a prism reflecting the degrees on the card so that its compass bearing can be easily read); then the true bearing of the sun at the same instant is found from the azimuth tables. The difference between the true bearing and the compass bearing is the total error. The variation found from the chart in the locality where the vessel was swung applied to the total error gives the deviation on that heading.

EXAMPLE 2:

Vessel's heading by standard compass	Watch time of observation	Correction watch on L. A. T.	L. A. T. of observation	Azimuth of sun by compass	True azimuth of sun by az. tables	Error	Variation from chart	Deviation
°	h m s		h m s	° ′	° ′	° ′		° ′
0	7 49 40	Error of	7 46 20	112 30	102 00	−10 30	10° W	−0 30
15	7 54 05	watch on	7 50 45	113 00	102 47	−10 13	or −10°	−0 13
30	7 58 35	L. A. T.	7 55 15	114 30	103 36	−10 54		−0 54
45	8 03 40	$3^m\ 20^s$ fast	8 00 20	116 30	105 15	−11 15		−1 15
and so on.								

In later chapters it will be shown how the deviation is checked by observations on the sun, moon, and stars, and kept correct on every course the vessel runs.

36. Leeway. — A steamer making a passage in calm weather or a ship with a fair wind, having their compass courses corrected for deviation and variation, makes a track through the water in a true direction, but when the wind blows on the side of the vessel, or in any direction except from dead ahead or astern it has a tendency to drive the vessel before it and the direction of the vessel's head is no longer the same as its track; the angular distance between the keel and the wake of the vessel is the leeway.

The amount of the leeway in degrees varies in different vessels, under different conditions, depending upon the build of the vessel and upon the force and direction of the wind. In modern practical navigation leeway is usually disregarded in steamers, unless the wind be blowing hard.

There is no real good way to measure leeway; it is simply estimated by the eye, by looking astern at the visible track of the vessel in the water, and depends for its correctness on the good judgment of the observer. Leeway is allowed away from the direction the wind blows; thus, with the wind on the starboard side of the vessel, or on the starboard tack in a sailing ship, a course is corrected to the left; on the port side it is corrected to the right, estimated on the supposition that the observer is at the center of the compass and looking in the direction the vessel is sailing or steaming.

EXAMPLE 1: The compass course of a vessel is 315° (N. W.), the wind is N. N. E., the estimated leeway is 22° 30′ (2 points), Dev. 3° W., Var. 15° E., find the true course.

Compass course	315	00	or Leeway	22	30 W.
Leeway	22	30 (left)	Dev.	3	00 W.
	292	30		25	30 W.
Dev.	3	00 W.	Var.	15	00 E.
Magnetic course	289	30	Total corr.	10	30 W.
Var.	15	00 E.	Compass course	315	00
True course	304	30	Error	10	30 W.
			True course	304	30

CHAPTER IV

POSITION BY OBSERVATION OF TERRESTRIAL OBJECTS

37. Methods of Fixing Position. — A navigator coasting and in sight of objects whose positions are shown on the chart may locate the vessel by various methods such as (*a*) cross bearings of two known objects; (*b*) the bearing and distance of a known object; (*c*) two bearings of a known object separated by an interval of time, with the run during that interval; (*d*) sextant angles between three known objects.

38. Position by Cross Bearings. — When a vessel's position is fixed by the intersection of two or more lines of bearing, the position is said to be fixed by cross bearings.

Two objects should be selected, the bearings of which give as near a right-angled cut as possible. The bearing of a third object should be taken when possible, not only because it is a check on the observations, but because it insures against the possibility of laying off a position line from the wrong point.

Objects abeam and before the beam should be used in preference to objects abaft the beam which the vessel has already passed.

Objects should be selected that are near the vessel, in preference to those far away, because any error in the bearing of a near object has less effect on the position than the same error in the bearing of a distant object; and because charts are somewhat distorted in printing, long lines of bearing are never so accurate as short ones.

When taking cross bearings they should be observed as quickly as possible, with due regard to accuracy — the object whose bearing is changing most rapidly being observed last. Should the objects be changing their bearings quickly, it is impossible to get a satisfactory cut if some time elapses between taking the bearings of the different objects.

39. Position by Bearing and Distance. — When only one object is in sight, its distance can be obtained either by range-finder or by vertical sextant angle. If the distance of an object can be obtained, the vessel must be located on the circumfer-

ence of a circle described, with the object as center and with the distance as radius. This circle is, therefore, the position line.

The distance from an object may be found in the following ways:

By a Rangefinder. — If the object is well defined, the rangefinder, provided it is in good adjustment, is the quickest and most accurate method of obtaining the distance, within the limits of the instrument.

By a Sextant. — When the height of the object is known, the distance can be found by measuring the angle with the sextant. At the same time take its bearing; the position of the vessel must be at the intersection of the line of bearing and the circle described, with the object as center and the distance as radius. This is an exceedingly useful method of fixing the position of the vessel quickly.

40. Two Bearings of a Known Object. — This is a most useful method, which is frequently employed, certain special cases arising thereunder being particularly easy of application. Take a careful bearing of the object and at the same moment read the patent log; then, after running a convenient distance, take a second bearing and again read the log, the difference in readings giving the intervening run; when running at a known speed, the time interval will also afford a means for determining the distance run.

The Bow and Beam or Four-point Bearing. — This is one of the most simple of the two bearings of a known object and a method used at sea probably more than any other. It is as follows: when the object bears by compass four points (45°) from the course, take the time by watch and read the log; when it bears on the beam, or 90° from the course, read the watch and log again. The distance run by the vessel in the interval is the distance away from the object when abeam.

EXAMPLE: A vessel on a course 130° takes the first bearing of a light at 160°, and the second at 184°, running in the interval 1.8 miles. Find the distance the vessel will pass abeam and also the distance of the light at the second bearing.

Difference between course and first bearing 30°.
Difference between course and second bearing, 54°.
(Tab. 5B. Bowd.) Multiplier from second column, 0.99.
1.8 miles × 0.99 = 1.8 miles distance of passing abeam.
(Tab. 5B.) Multiplier from first column, 1.23.
1.8 miles × 1.23 = 2.2 miles distance at 2d bearing.

41. Position by Horizontal Sextant Angles. — The method of fixing the vessel's position by the intersection of two or more position lines, obtained by observing the horizontal sextant angles subtended by three or more objects, is extremely useful in navigation, when great accuracy is essential or when no compass is available, as the observations can be taken from any part of the vessel.

The position is more exact than the position found by bearings, because the angles can be measured with a sextant with greater precision than the compass will permit and there is no correction for deviation or variation. This method is especially valuable when the objects available are at a considerable distance from the observer, when the drawing of long lines of bearing introduces error which is difficult to avoid. Another advantage is that the observer is not tied to any one spot, and can therefore place himself in the most advantageous position to see the objects clear of masts or other obstacles which ordinarily obscure so many points. The position of the vessel is plotted with the three-arm protractor.

CHAPTER V

THE SAILINGS

PLANE SAILING

42. Suppose the compass needle constantly to point to the north, a vessel which is steered by it upon any given course must *cross every meridian at the same angle;* namely, the angle given by the compass. The vessel does not steam on a great circle, except when on the equator, east or west, or on a meridian, north or south. All other great circles intersect successive meridians at varying angles.

A line which makes the same angle with each successive meridian is called a *rhumb-line;* more commonly, the *vessel's track*, or a *loxodromic curve.*

The constant angle which it makes with the meridian is the *course*, and is called the *true course*, to distinguish it from the *compass course.*

The length of the line considered, or the distance sailed, is called the *distance.*

The corresponding increase or decrease of latitude is the *difference of latitude.*

The distance between the meridian left, and that arrived at, measured on a parallel of latitude, is the *departure* on that parallel.

The distance between these meridians, measured on the equator, is the corresponding *difference of longitude.*

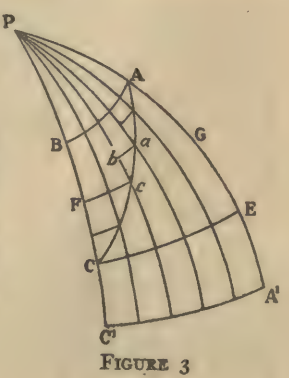


Figure 3

43. The following notation will be employed, the references being to Fig. 3, in which C A represents a portion of a loxodromic curve:

C = B C A, the course.

d = C A, the distance.

l = C B = E A, the difference of latitude.
p = the departure: C E in the latitude of C, B A in the latitude of B, F G in the latitude of F.
D = C′ A′, the difference of longitude.
L = C′ C, the latitude left, } + when north.
L' = A′ A, the latitude arrived at, } − when south.
λ = the longitude left, } + when west.
λ' = the longitude arrived at, } − when east.

$$\left.\begin{array}{lll} \text{Evidently} & l = L' - L, & D = \lambda' - \lambda; \\ \text{whence} & L' = L + l, & \lambda' = \lambda + D, \end{array}\right\} \quad (1)$$

in which attention must be paid to the signs, or names.*

44. If the distance is very small, so that the curvature of the earth may be neglected, then C A may be regarded as a right line, and the triangle C A B as a right plane triangle. From this we have

$$\cos C = \frac{l}{d}, \qquad \sin C = \frac{p}{d}, \qquad \tan C = \frac{p}{l}; \quad (2)$$

or

$$l = d \cos C, \qquad p = d \sin C, \qquad p = l \tan C, \quad (3)$$

in which p is the departure in the latitude of C or A; indifferently, as their distance is very small.

The Traverse Table, or Table of Right Triangles, contains l and p for different values of C and d. Table 1 in Bowditch's "Navigator" contains l and p for each unit of d from 1 to 300, and for each quarter-point of C. Table 2 contains them for each unit of d and each degree of C.

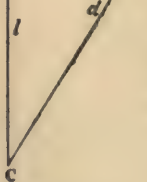


FIGURE 4

These quantities form a plane right triangle (Fig. 4), in which

d is the hypothenuse,
C one of the angles,
l the side adjacent } that angle.
p the side opposite }

In the Tables, the columns of *distance*, *difference of latitude*, and *departure* might be appropriately headed, respectively, *hypothenuse*, *side adjacent*, and *side opposite*.

*If N. and W. are regarded as positive, S. and E. are negative, and may be treated as such, without the formality of substituting the signs + and −.

45. The first two of equations (3) afford the solution of the most common elementary problem of navigation and surveying, viz.:

Given the course and distance, to find the difference of the latitude and departure, the distance being so small that the curvature of the earth may be neglected.

These equations also afford solutions of all the cases of Plane Sailing.

46. ***Given the course and distance, to find the difference of latitude and departure, when the distance is so great that the curvature of the earth cannot be neglected.***

Solution. Let the distance C A (Fig. 3) be divided into parts, each so small that the curvature of the earth may be neglected in computing its corresponding difference of latitude and departure. For each such small distance, as $c\,a$,

$$l = d \cos C, \qquad p = d \sin C.$$

Representing the several partial distances by d_1, d_2, d_3, etc., the corresponding values of l and p by l_1, l_2, l_3, etc., and p_1, p_2, p_3, etc., and the sums respectively by $[d]$, $[l]$, $[p]$, we have

$$l_1 + l_2 + l_3 + \text{etc.} = (d_1 + d_2 + d_3, \text{etc.}) \cos C,$$
$$p_1 + p_2 + p_3 + \text{etc.} = (d_1 + d_2 + d_3, \text{etc.}) \sin C;$$

or,
$$[l] = [d] \cos C, \qquad [p] = [d] \sin C.$$

Since the distance between two parallels of latitude is the same on all meridians, the sum of the several partial differences of latitude will be the whole difference of latitude; as in Fig. 3.

C B = E A = the sum of all the sides, $c\,b$, of the small triangles;

and we shall have generally, as in Art. 45,

We also have
$$l = d \cos C.$$
$$p = d \sin C,$$

if we regard p as the *sum of the partial departures*, each being taken in the latitude of its triangle; so that the difference of latitude and departure are calculated by the same formulas, when the curvature of the earth is taken into account, as

when the distance is so small that the curvature may be disregarded; or, in other words, *as if the earth were a plane.*

But the sum of these partial departures, $b\,a$ of Fig. 3, is evidently less than C E, the distance between the meridians left and arrived at on the parallel C E, which is nearest the equator; and greater than B A, the distance of these meridians on the parallel B A, which is farthest from the equator. But it is *nearly* equal to F G, the distance of these meridians on a middle parallel between C and A; and we take then $L_0 = \frac{1}{2}\ (L + L')$, as the latitude for the departure, p.

47. Middle Latitude Sailing regards the departure, p, as the distance between the meridian left and that arrived at on the middle parallel of latitude; or takes $L_0 = \frac{1}{2}\ (L + L')$.

TRAVERSE SAILING

48. If the ship sail on several courses, instead of a single course, she describes an irregular track, which is called a *Traverse.*

To reduce several courses and distances to a single course and distance, and find the corresponding differences of latitude and departure.

Solution. If in Fig. 3 we regard C as different for each partial triangle, and represent the several courses by C_1, C_2, C_3, etc., we evidently have

$$l_1 = d_1 \cos C_1, \qquad p_1 = d_1 \sin C_1, \ ;$$
$$l_2 = d_2 \cos C_2, \qquad p_2 = d_2 \sin C_2,$$
$$l_3 = d_3 \cos C_3, \text{ etc.} \qquad p_3 = d_3 \sin C_3, \text{ etc.}$$

and $[l] = l_1 + l_2 + l_3$, etc., $\qquad [p] = p_1 + p_2 + p_3$, etc.;

or, as in the more simple case of a single course,

The whole difference of latitude is equal to the sum of the partial differences of latitude;

The whole departure is equal to the sum of the partial departures.

This applies to all cases, if we use the word *sum* in its general or algebraic sense.

If we represent by L_n the sum of the northern diffs. of latitude,
" " L_s " southern " "
" " P_w " western departures,
" " P_e " eastern "

we have as the *arithmetical* formulas,

$[l] = L_n \sim L_s$ of the same name as the greater,
$[p] = P_w \sim P_e$ " " " " "

which accord with the usual rules. (Bowditch, Arts. 172, 205.)*

The *Traverse Form* (pp. 26 and 27) facilitates the computation.

The course, C, and distance $[d]$, corresponding to $[l]$ and $[p]$, may be found *nearly* by Plane Sailing.†

49. The departure may be regarded as measured on the middle parallel, either between the extreme parallels of the traverse, or between that of the latitude left and that arrived at. In a very irregular traverse it is difficult to determine the precise parallel; but, except near the pole, and for a distance exceeding an ordinary day's run, the middle latitude suffices.

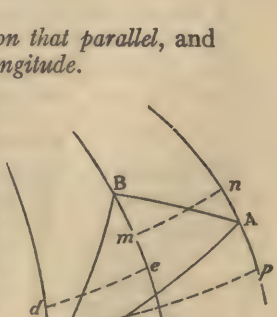
It is easy, however, to separate a traverse into two or more portions, and compute for each separately.

PARALLEL SAILING

50. The relations of the quantities C, d, l, and p are expressed in equations (3). When the difference of longitude also enters, then some further considerations are necessary, since the earth's surface must now be regarded, not as plane, but spherical.

To find the relations between —

L, the latitude of a parallel,
p, the departure of two meridians on that parallel, and
D, the corresponding difference of longitude.



* These references are to Bowditch, "American Practical Navigator," where other reading on the subject is given, but it is not necessary to consult this book.

† C and $[d]$ are not accurately found, because $[p]$, the sum of the partial departures of the traverse, is not the same as p, the departure of the loxodromic curve connecting the extremities of the traverse. Thus, suppose a ship to sail from C to A by the traverse C B, B A, her departure will be by traverse sailing $d\,e + m\,n$; whereas, if the ship sail directly from C to A, the departure will be $o\,p$, which is greater or less than $d\,e + m\,n$, according as it is nearer to, or farther from the equator. Thus we should obtain in the two cases a different course and distance between the same two points. In ordinary practice, however, such difference is immaterial.

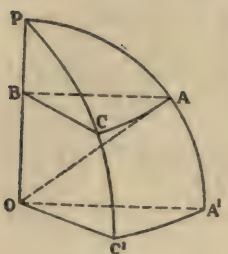


FIGURE 5

Solution. In Fig. 5, let
P A A′, P C C′ be two meridians.
A C = p, their departure on the parallel A C, whose latitude is A O A′ = O A B = L, and whose radius is B A = r.
A′ C′ = D, the measure of A P C, the difference of longitude of the same meridians, on the equator A′ C′, whose radius is O A′ = O A = R.

A C, A′ C′ are similar arcs of two circles, and are therefore to each other as the radii of those circles; that is,

A C : A′ C′ = B A : O A′, or $p : D = r : R$.

In the right triangle O B A,

$$B A = O A \times \cos O A B, \quad \text{or} \quad r = R \cos L; \tag{4}$$

that is, *the radius of a parallel of latitude is equal to the radius of the equator multiplied by the cosine of the latitude.*

Substituting (4) in the preceding proportion, we obtain

$$p : D = \cos L : 1,$$

or

$$p = D \cos L, \qquad D = p \sec L, \tag{5}$$

which express the relations required. (Bowditch, Art. 173.)

These relations may be graphically represented by a right plane triangle (Fig. 6), of which

D is the hypothenuse,
L, one of the angles,
p, the side adjacent that angle.

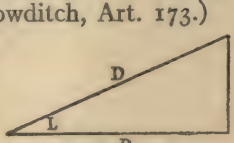


FIGURE 6

The Traverse Table, or Table of Right Triangles, may therefore be used for the computation.

MIDDLE LATITUDE SAILING

51. ***Given the course and distance and the latitude left, to find the difference of longitude.***

Solution. By Plane Sailing.

$$l = d \cos C, \qquad p = d \sin C; \tag{3}$$

by Arts. 43 and 47,

$$L' = L + l, \qquad L_0 = \tfrac{1}{2}(L' + L) = L + \tfrac{1}{2}l; \tag{6}$$

and by equation (5),

$$D = p \sec L_0, \tag{7}$$

or

$$D = d \sin C \sec L_0. \tag{8}$$

Equations (3), (6), and (7) or (8) afford the solution required.

The assumption of $L_0 = \frac{1}{2}(L' + L)$, or the *middle latitude*, suffices for the ordinary distance of a day's run; but for larger

distances, and where precision is required, we should use "Mercator's Sailing" (Art. 57).

52. Strictly, the middle latitude should be used only when both latitudes, L and L', are of the same name, as is evident from Fig. 3.

If these latitudes are of different names, and the distance is small, $\frac{1}{2}(L + L')$, numerically, may be used; or we may even take $p = D$, since the meridians near the equator are sensibly parallel.

If the distance is great, the two portions of the track on different sides of the equator may be treated separately. (Art. 61.)

When several courses and distances are sailed, as is ordinarily the case in a day's run, p and l are found as in Traverse Sailing, and then D by regarding p as on some parallel midway between the extremes of the traverse. (Art. 49) (Bowditch, Art. 205.)

53. The relations of the quantities involved in Middle Latitude Sailing, namely,

C, d, p, l, L_0, and D,

are represented graphically by combining the two triangles of Plane Sailing and Parallel Sailing, as in Fig. 7, in which

$C = \text{A C B}$, $p = \text{B A}$, $L_0 = \text{B A E}$,
$d = \text{C A}$, $l = \text{C B}$, $D = \text{A E}$,

By these two right triangles, all the common cases classed under Middle Latitude Sailing (Bowditch, Art. 176) may be solved, if we add the formulas,

$$L' = L + l \qquad \lambda' = \lambda + D.$$

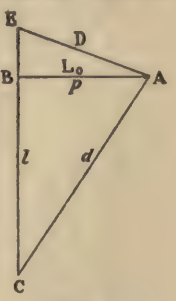


FIGURE 7

EXAMPLE IN MIDDLE LATITUDE SAILING

EXAMPLE 1: Find the course and distance from San Francisco, Lat. 37° 48′ N., Lon. 122° 28′ W., to Yokohama, Lat. 35° 26′ N. Lon. 139° 39′ E.

$L_1 = 37° 48'$ N.	$\lambda_1 = 122° 28'$ W.	$L_1 = 37° 48'$
$L_2 = 35° 26'$ N.	$\lambda_2 = 139° 39'$ E.	$L_2 = 35° 26$
$l = 2° 22'$ S. $= 142'$	D = 97° 53′ W. = 5873	$2)73° 14$
		$L_0 = 36° 37'$

$D = 5873$	log 3.76886		
$L_0 = 36° 37'$	cos 9.90452		
p	log 3.67338	log 3.67338	
l		log 2.15229	log 2.15229
$C = $ S. 88° 16′ W. (268° 16′)		tan 1.52109	sec 1.52126
$d = 4715.8$ miles			log 3.67355

EXAMPLES IN TRAVERSE SAILING

54. EXAMPLE: June 3, 1918, a vessel under sail heading NE. $\frac{3}{4}$ E. (deviation $\frac{1}{4}$ pt. East) takes departure from Cape Henry lighthouse (Lat. 36° 55′ 35″ N., Lon. 76° 00′ 27″ W.), bearing SSW. $\frac{1}{2}$ W. by compass, distant 1.4 miles. Then sails the courses, with errors and distances, as given below; wind SE. by E. Find the position by dead reckoning; also course and distance made good. (Table 1, Bowditch.)

Comp. course	Var.	Dev.	Leeway	Error	True course	Dist.	N.	S.	E.	W.
NNE. $\frac{1}{2}$ E.	$\frac{1}{2}$ W.	$\frac{1}{4}$ E.		$\frac{1}{4}$ W.	NNE. $\frac{1}{4}$ E.	1.4	1.3		0.6	
NE. $\frac{3}{4}$ E.	$\frac{1}{2}$ W.	$\frac{1}{4}$ E.	$\frac{1}{4}$ W.	$\frac{1}{2}$ W.	NE. $\frac{1}{4}$ E.	27.6	18.5		20.5	
S. by W.	$\frac{1}{2}$ W.	0	$\frac{1}{4}$ E.	$\frac{1}{4}$ W.	S. $\frac{3}{4}$ W.	31.5		31.2		4.6
ENE.	$\frac{1}{2}$ W.	$\frac{1}{4}$ E.	$\frac{1}{2}$ W.	$\frac{3}{4}$ W.	NE. by E. $\frac{1}{4}$ E.	14.2	7.3		12.2	
S. $\frac{1}{4}$ E.	$\frac{1}{2}$ W.	0	$\frac{1}{2}$ E.	0	S. $\frac{1}{4}$ E.	11.0		11.0	0.5	
NE. $\frac{1}{4}$ N.	$\frac{3}{4}$ W.	$\frac{1}{4}$ E.	$\frac{1}{4}$ W.	$\frac{3}{4}$ W.	NE. by N.	87.0	72.3		48.3	
							99.4	42.2	82.1	4.6
						96.5		p =	77.5	
C, made good,					NE. $\frac{3}{4}$ E.	l =	57.2		D =	97.0

	Latitude.		*Longitude.*
Point of departure,	36° 55′.6 N.		76° 00′.5 W.
l =	57.2 N.	$L_0 = 37°$	1 37.0 E. = D
By D. R.	37 52.8 N.		74 23.5 W.

EXAMPLE 2: April 23, 1918, at noon, in Lat. 41° 31′ 00″ N., Lon. 178° 25′ 00″ W., by observation, a vessel then steams the following true courses until 7 A.M., April 25, find the position by dead-reckoning.

Course	Dist.	N.	S.	E.	W.
201°	29	—	27.1	—	10.4
217°	20.6	—	16.5	—	12.4
236°	72	—	40.3	—	59.7
251°	16.4	—	5.3	—	15.5
262°	23.7	—	3.3	—	23.5
272°	45	1.6	—	—	45.0
		1.6	92.5	p =	166.5
		l =	90.9	D =	219.8

Lat. by obs. 41° 31′ 00″ N. L_0 = 40° 45′ 33″ Lon. by obs. 178° 25′ 00″ W.
l = 1° 30′ 54″ S. D = 3° 39′ 48″ W.
Lat. by D. R. 40° 00′ 06″ N. 177° 55′ 12″ E.

55. *Current by difference between dead-reckoning and observed positions.* — If the position of the vessel is found by observation to differ from the D.R. position, it is obvious that some of our data are incorrect. As it is impossible to determine which of the data are wrong, the difference between the observed and the D.R. positions is attributed to the current alone, because this generally has the greatest effect in displacing the vessel. For instance, in the preceding example, suppose that the observed position of the vessel at 7 A.M. was found to be Lat. 40° 03′ N., Lon. 178° 07′ E. The difference between the D.R. position and this observed position is expressed by finding the course and distance from the former to the latter, and assuming that this course and distance were due to the set and drift of the current in the interval.

D.R. position	Lat. 40° 00′ N.,	Lon. 177° 55′ E.
Observed position	Lat. 40 03 N.,	Lon. 178 07 E.
	l = 3 N.,	D = 12 E.

With L_0 = 40° N., and D = 12′ E., we find p = 9.2 E. With l = 3′ N., and p = 9.2 E., we find the course and distance to be N. 72° E., 9.7 miles, which gives the set and drift of the current as N. 72° E. (true) 0.5 knots.

CURRENT SAILING

56. *When finding the course to allow for a current.* — When a vessel's motion is influenced by currents or tidal streams, her direction of movement is not, in general, the same as that of the keel. The direction of the vessel's track at any time is called the course made good, and the actual speed over the bottom of the sea in that direction is called the speed made good; the latter is sometimes referred to as the speed over the ground, in distinction to the speed through the water.

The direction in which a current is running is called the *set* of the current, and the speed in knots at which it is running is called the *drift* of the current. The set and drift of a current may be obtained from a pilot chart or a current chart.

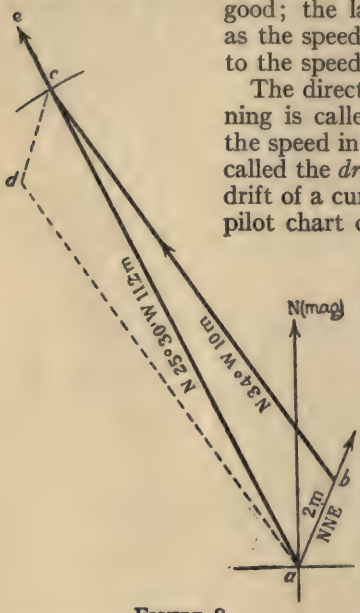


FIGURE 8

To find what course is steered in order that the course made good is all right, the law of physics of the composition of velocities is employed.

EXAMPLE 1: Suppose that a vessel's magnetic course is N. 25° 30′ W. From the pilot or other chart it is seen that a current running NNE. (mag.) 2 knots will probably be encountered. It is required to find the compass course. From a, Fig. 8, lay off $a\,b$, to represent NNE. 2 knots (the set and drift of the current expected) on any convenient scale. With center b and radius $b\,c$ to represent 10 knots (the vessel's speed through the water on the same scale) describe a circle cutting $a\,e$ in c; then the direction of $b\,c$ which is N. 34° W. (mag.) is the course to make good N. 25° 30′ W. (mag.) on the assumption that the set and drift of the current is NNE. 2 knots.

Magnetic course	N. 34° W.
Deviation (from vessel's deviation table)	2° E.
Compass course	N. 36° W.

The speed made good along the line $a\,e$ is the length $a\,c$ 11.2 knots.

MERCATOR'S SAILING

57. Middle Latitude Sailing suffices for the common purposes of navigation as a day's run, but a more rigorous solution of problems relating to the rhumb-line is needed for finding the course between two ports separated widely in latitude, or for determining the difference of longitude of these places. These solutions come under "Mercator's Sailing."

A ship sails from the equator on a given course, C, till she arrives in a given latitude, L; to find the difference of longitude, D.

Solution. Let the sphere (Fig. 9) be projected upon the plane of the equator stereographically. The primitive circle A B C. . . . M is the equator.

P, its center, is the pole (the eye or projecting point being at the other pole.)*

The radii, P A, P B, P C, etc., are meridians making the same angle with each other in the projection as on the surface of the sphere.*

The distance P m, of any point m from the center of the projection, = tan $\frac{1}{2}$ (90°− L), the tan gent of $\frac{1}{2}$ the polar distance of the point on the surface which m represents, the radius of the sphere being 1.*

This curve in projection makes the same angle with each meridian as the loxodromic curve with each meridian on the surface.*

A M is the whole difference of longitude D.

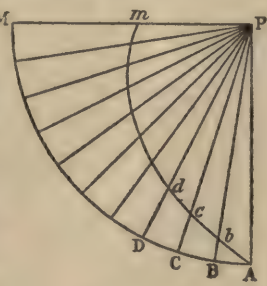


FIGURE 9

If we suppose this divided into an indefinite number of equal parts, A B, B C, C D, etc., each indefinitely small, and the meridians P A, P B, P C, etc., drawn, the intercepted small arcs of the curve A $b\,c$ . . . m may be regarded as straight lines, making the angles P A b, P $b\,c$, P $c\,d$, etc., each equal to the course C; and consequently the triangles P A b, P $b\,c$, P $c\,d$, etc., similar. We have then

$$\text{P A} : \text{P } b = \text{P } b : \text{P } c = \text{P } c : \text{P } d, \text{ etc.},$$

* Principles of stereographic projection.

or the geometrical progression,

$$\text{P A} : \text{P } b : \text{P } c : \ldots . \text{P } m.$$

If then

D = the whole difference of longitude,
d = one of the equal parts of D,
$\frac{D}{d}$ will be the number of parts, and
$\frac{D}{d} + 1$ the number of meridians P A, P b . . . . P m,

or the number of terms of the geometrical progression: and, employing the usual notation,

the first term $a = \text{P A} = 1$,
the last term $l = \text{P } m = \tan \frac{1}{2} (90° - L)$,

$$n - 1 = \frac{D}{d},$$

the ratio

$$r = \frac{\text{P } b}{\text{P A}}.$$

To find this ratio, we have in the indefinitely small right triangle A B b,

$$\tan \text{B A } b = \cot \text{P A } b = \frac{\text{B } b}{\text{B A}},$$

or

$$\cot C = \frac{\text{P A} - \text{P } b}{d},$$

whence

$$\text{P A} - \text{P } b = d \cot C;$$
$$\text{P } b = \text{P A} - d \cot C,$$

and, since P A = 1,

$$r = \frac{\text{P } b}{\text{P A}} = 1 - d \cot C.$$

Then by the formula for a geometrical progression,

$$l = a r^{n-1},$$

By substituting the value given above we have

$$\tan \tfrac{1}{2} (90° - L) = (1 - d \cot C)^{\frac{D}{d}}.$$

Taking the logarithm of each member, we have

$$\log \tan \tfrac{1}{2} (90° - L) = \frac{D}{d} \log (1 - d \cot C). \qquad (9)$$

But we have in the theory of logarithms

(*Naperian*) $\log (1 + n) = n - \frac{n^2}{2} + \frac{n^3}{3} - \frac{n^4}{4} + \text{etc.} \ldots .$

and

$$(\textit{Common})\ \log (1 + n) = m\left[n - \frac{n^2}{2} + \frac{n^3}{3} - \frac{n^4}{4} + \text{etc.} \ldots .\right] \quad (10)$$

in which the modulus $m = .434294482$.

Hence, putting $n = -d \cot C$,

$$\log (1 - d \cot C) = m\left[-d \cot C - \tfrac{1}{2} d^2 \cot^2 C - \tfrac{1}{3} d^3 \cot^3 C - \text{etc.} \ldots .\right],$$

and substituting in (9) and reducing,

$$\log \tan \tfrac{1}{2} (90^\circ - L) = -m \times D\left[\cot C + \tfrac{1}{2} d \cot^2 C + \tfrac{1}{3} d^3 \cot^3 C + \text{etc.} \ldots .\right]. \quad (11)$$

This equation is the more accurate the smaller d is taken, so that if we pass to the limit and take $d = 0$, it becomes perfectly exact. The broken line A b c . . . . m then becomes a continuous curve, and our equation (11) becomes

$$\log \tan \tfrac{1}{2} (90^\circ - L) = -m \times D \cot C;$$

whence

$$D = -\frac{\log \tan \tfrac{1}{2} (90^\circ - L)}{m} \tan C. \quad (12)$$

But in this equation D is expressed in the same unit as tan C, that is, in *terms of radius*. (Chauvenet's Plane Trigonometry.)

To reduce it to minutes we must multiply it by the radius in minutes, or $r' = 3437'.74677$.

Substituting the value of m, we shall have (in minutes),

$$D = -\frac{3437'.74677}{.434294482} \log \tan \tfrac{1}{2} (90^\circ - L) \tan C.$$

To avoid the negative sign, we observe that

$$\tan \tfrac{1}{2} (90^\circ - L) = \frac{1}{\cot \tfrac{1}{2} (90^\circ - L)} = \frac{1}{\tan \tfrac{1}{2} (90^\circ + L)},$$

or that

$$-\log \tan \tfrac{1}{2} (90^\circ - L) = \log \tan \tfrac{1}{2} (90^\circ + L).$$

Hence we have, by reducing,

$$D = 7915'.70447 \log \tan (45^\circ + \tfrac{1}{2} L) \tan C. \quad (13)$$

NOTE.— This may be more readily solved, and equation (13) obtained by aid of the Calculus, as follows:

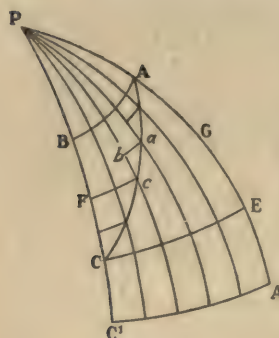


FIGURE 1

In Fig. 1, suppose $c\ a$ to be an element of the loxodromic curve C A:
$c\ b$ will be the corresponding element of the meridian, and
$b\ a \times \sec L$, the element of the equator;
L being the latitude of the indefinitely small triangle $c\ a\ b$.

By Articles 46 and 51, using the notations of the Calculus, we have

$$\mathrm{d}L = \cos C\, \mathrm{d}d \qquad \mathrm{d}p = \tan C\, \mathrm{d}L$$
$$\mathrm{d}D = \sec L\, \mathrm{d}p = \tan C \sec L$$

in which C is constant.

By integrating the last equation between the limits $L = 0$ and $L = L$, we shall have

$$D = \tan C \int_0^L \sec L\, \mathrm{d}L,$$

the whole difference of longitude required in Art. 57.

To effect the integration, put

$$\sin L = x, \qquad \text{then by differentiating,}$$

$$\mathrm{d}L = \frac{\mathrm{d}x}{\cos L}, \qquad \text{and multiplying by sec } L,$$

$$\sec L\, \mathrm{d}L = \frac{\mathrm{d}x}{\cos^2 L} = \frac{\mathrm{d}x}{1 - \sin^2 L}, \text{ or}$$

$$\sec L\, \mathrm{d}L = \frac{\mathrm{d}x}{1 - x^2}.$$

Resolving into partial fractions, we obtain

$$\sec L\, \mathrm{d}L = \tfrac{1}{2}\left[\frac{\mathrm{d}x}{1+x} + \frac{\mathrm{d}x}{1-x}\right] \text{ and}$$

$$\int_0^L \sec L\, \mathrm{d}L = [\log (1+x) - \log (1-x)]$$

$$= \log \sqrt{\frac{1+x}{1-x}}$$

$$= \log \sqrt{\frac{1+\sin L}{1-\sin L}}$$

$$= \log \tan (45^\circ + \tfrac{1}{2} L) \text{ (Plane Trigonometry.)}$$

Whence we have $D = \log \tan (45^\circ + \frac{1}{2} L) \tan C.$

But in this the logarithm is Napierian and D is expressed in terms of the radius of the sphere. To reduce to common logarithms, we divide by $m = .434294482$, and to minutes by multiplying by $r' = 3437'.74677$, and obtain

$$D = 7915'.70447 \log \tan (45^\circ + \tfrac{1}{2} L) \tan C,$$

as in (13).

58. Formula (13) is based upon the assumption that the earth is a sphere. Allowing for the meridional eccentricity of the earth, which according to Clarke is

$$\frac{1}{293.465} = 0.003407562 = c,$$

the formula used for computing the table of meridional parts is

$$M = 7915'.7045 \log \tan (45^\circ + \tfrac{1}{2} L) - a\,(e^2 \sin L + \tfrac{1}{3} e^4 \sin^3 L + \tfrac{1}{5} e^6 \sin^5 L + \ldots .), \quad (14)$$

in which $e = \sqrt{2c - c^2} = 0.0824846$

and $ae^2 = 23'.38871$ $\tfrac{1}{3} ae^4 = 0'.053042$ $\tfrac{1}{5} ae^6 = 0'.000216523$

The values of M computed by (14) for each minute of L from 0° to 79° 59′ form the Table of Meridional Parts or Augmented Latitudes. (Bowditch, Table 3.)

In practice, then, we have only to take the value of M corresponding to L, and D is then found by the formula,

$$D = M \tan C. \quad (15)$$

M has the same name, or sign, as L.

59. ***A ship sails from a latitude, L, to another latitude, L′, upon a given course, C; find the difference of longitude, D.***

Solution. Let

M be the augmented latitude corresponding to L,
M' " " " " L'.

The difference of longitude from the point, A, where the track crosses the equator to the first position, whose latitude is L, will be

$$D_l = M \tan C;$$

and to the second position, whose latitude is L',

$$D_{ll} = M' \tan C;$$

and we shall have

$$D = D_{ll} - D_l = (M' - M) \tan C; \quad (16)$$

or, when $M' < M$,

$$D = D_l - D_{ll} = (M - M') \tan C;$$

since the sign of D is determined by the course.

If L and L' are of different names, so also are M and M', and we have *numerically*

$$D = (M + M') \tan C.$$

60. The difference, $M' - M$, is called the *meridional*, or *augmented, difference of latitude*. Representing this by m, we have

$$D = m \tan C. \qquad (17)$$

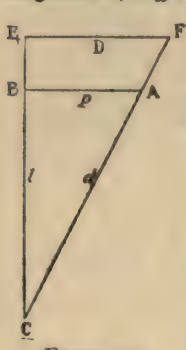


FIGURE 10

The relation between these quantities is represented by a plane right triangle (Fig. 10), in which

C is one of the angles,
m = C E, the side adjacent,
D = E F, the side opposite.

The triangle of "Plane Sailing" has the same angle C, with

l = C B, the adjacent side,
and p = B A, the opposite side.

Fig. 10 represents these two triangles combined. By them, all the common cases under Mercator's Sailing can be solved, either by computation or by the Traverse Table.

The relations between the several parts involved are

$$\left.\begin{array}{ll} l = d\cos C, & L' = L + l, \\ p = d\sin C, & m = M' - M, \\ D = m\tan C, & \lambda' = \lambda + D; \end{array}\right\} \qquad (18)$$

and since $p = l \tan C$,
$l : m = p : D.$

61. ***Given the latitudes and the longitudes of two places, find the course, distance, and departure.*** (BOWDITCH Art. 182.)

Solution. L and L' being given, we take from Table 3 M and M'.

We have $l = L' - L$, $\quad m = M' - M$, $\quad D = \lambda' - \lambda$;

by Mercator's Sailing, $\tan C = \dfrac{D}{m}$;

and by Plane Sailing, $d = l \sec C$, $\quad p = l \tan C$;

l, m, and C are *north* or *south* according as L' is *north* or *south* of L.

D, p, and C are *east* or *west*, according as λ' is *east* or *west* of λ.

If the two places are on opposite sides of the equator, we have *numerically*

$$l = L' + L, \qquad m = M' + M.$$

EXAMPLE 1: Find the course and distance from Cape Frio, Lat. 23° 01′ S., Lon. 42° 00′ W., to Lizard Point, Lat. 49° 58′ N., Lon. 5° 12′ W.

L = 23° 01′ S.	M = 1409.5 S.	λ = 42° 00′ W.
L' = 49° 58′ N.	M' = 3453.4 N.	λ' = 5° 12′ W.
l = 72° 59′ N.	m = 4862.9 N.	D = 36° 48′ E = 2208′ E.
= 4379′		

D = 2208′ ..... log 3.34400
m = 4862.9 ..... log 3.68689
C N. 24° 25′ 15. E. ..... tan 9.65711 ..... sec 0.04070
l = 4379′ ..... log 3.64137
d = 4809.2 miles ..... log 3.68207

EXAMPLE 2: Find the course and distance from San Francisco, Lat. 37° 48′ N., Lon. 122° 28′ W. to Yokohama, Lat. 35° 26′ N., Lon. 139° 39′ E.

L = 37° 48′ N.	M = 2438.7	λ = 122° 28′ W.
L' = 35° 26′ N.	M' = 2262.5	λ_1 = 139° 39′ E.
l = 2° 22′ S.	m = 176 .2	D = 97° 53′ W. = 5873′
= 142′		

D = 5873′ ..... log 3.76886
m = 176.2 ..... colog 7.75399
C. S. 88° 17′ W. ..... tan 1.52285 ..... sec 1.52313
l = 142′ ..... log 2.15229
d = 4736.1 miles 3.67542

62. The rhumb-line or loxodromic curve on the surface of the earth and its stereographic projection (Fig. 9) present a peculiarity worthy of notice. Excepting a meridian and parallel of latitude, a line which makes the same angle with all the meridians which it crosses would continually approach the pole, until, after an indefinite number of revolutions, the distance of the spiral from the pole would becomes less than any assignable quantity. It is usual to say that such a curve meets the pole after an infinite number of revolutions. Still, however, it is limited in length.

For we have for the length of any portion,

by Plane Sailing, $d = (L' - L) \sec C$.

If $L = 0$ and $L' = 90° = \frac{\pi}{2}$,

the whole spiral from the equator to the pole will be, with radius = 1,

$$d = \frac{\pi}{2} \sec C.$$

If $L = -90° = -\frac{\pi}{2}$, and $L' = 90° = \frac{\pi}{2}$,

we have, as the entire length from pole to pole,

$$d = \pi \sec C.$$

If also $C = 0$, or the loxodromic curve is a meridian, $d = \pi$, a semicircumference, as it should be.

So also the length of the projected spiral $A\ b\ c$ . . . . (Fig. 9) from A to m can readily be shown to be (calling this length δ);

$$\delta = M\ m \sec C = [1 - \tan (45° - \tfrac{1}{2} L)] \sec C,$$

or (Plane Trigonometry)

$$\delta = \frac{2 \tan \frac{1}{2} L \sec C}{1 + \tan \frac{1}{2} L};$$

and its length from the equator to the pole, taking $L = 90°$,

$$\delta = \sec C.$$

A MERCATOR'S CHART

63. On a Mercator's chart, the equator and parallels of latitude are represented by parallel straight lines; and the meridians also by parallel straight lines at right angles with the equator. Two parallels of latitude, usually those which bound the chart, are divided into *equal parts*, commencing at some meridian and using some convenient scale to represent degrees, and subdivided to 10′, 2′, 1′, or some other convenient part of a degree, according to the scale employed.

Two meridians, usually the extremes, are also divided into degrees, and subdivided like the parallels of latitude, but by a scale increasing constantly with the latitude: so that any degree of latitude on such meridian, instead of being equal to a degree of the equator, is the *augmented degree*, or augmented difference of 1° of latitude, derived from a table of "meridional parts." (Bowditch, Table 3.) The meridian is graduated most conveniently by laying off from the equator the *augmented latitudes;* or from some parallel, the *augmented difference of latitude* for each degree and part of a degree, — using the same scale of equal parts as for the equator.

64. As on other maps and charts, parallels of latitude and meridians are drawn at convenient intervals; places, shore lines of continents and islands, harbors and rivers, etc., are

plotted, each point in its proper position; and such configurations of the land represented as the purpose of the chart requires.

65. *To plot on a chart a point whose latitude and longitude are given.* By means of the scales at the sides, draw a parallel of latitude in the latitude, and by means of the scales at the top or bottom, a meridian in the longitude of the point; or so much of each as suffices to find their intersection.

66. On nautical charts the soundings are shown in feet or fathoms, referred to low water datum, and also the character of the bottom; and on those of a large scale the contour lines of the bottom, or lines of equal depth, are shown from 10 to 100 fathoms. The variation of the compass and its annual rate of change is given at convenient intervals, also the lines of equal variation.

67. The meridians on this chart being parallel, arcs of parallels of latitude are represented as equal to the corresponding arcs of the equator: thus each is expanded in the proportion of the secant of its latitude to 1; as is evident from the formula

$$D = p \sec \mathrm{L}.$$

It can be shown that very small portions of the meridians are expanded in the same proportion; as for example, a degree whose middle latitude is 60° is 120′, or,

$$60' \text{ of the equator} \times \sec 60°.$$

But the two half degrees are unequally expanded; for

from $59\frac{1}{2}°$ to 60° is represented by 59′,
" 60° to $60\frac{1}{2}°$ " " 61′, nearly.

A small circle on the surface of the earth of 1° diameter at the equator is then represented by a circle, whose diameter is 1°;

in lat. 30° nearly by a circle, whose diameter is $1° \times \sec 30°$,
" 60° " " " " " $1° \times \sec 60°$,
" L " " " " " $1° \times \sec L$;

but not exactly by a circle, since the meridians are augmented more rapidly as the latitude is greater.

Such a chart, then, while representing a narrow belt at the equator in proper proportions, presents a view of the earth's surface expanded at each point, both in latitude and longitude, proportionally to the secant of its latitude.

68. If we take any two points, C F, on this chart, and join them by a straight line, and form a right triangle by a meridian through one, and a parallel of latitude through the other, we shall have the triangle of Mercator's sailing (Fig. 11): for the intercepted portion of the meridian, C E, is the augmented difference of latitude; and of the parallel of latitude, E F is the difference of longitude. Hence the angle E C F is the course. (Art. 60.) Moreover, the rhumb-line is represented by the straight line C F; for if we take any intermediate point of this curve, and let *d* be its position on the chart, *d* must be in the line C F, otherwise when we construct the triangle of Mercator's Sailing we shall have an angle at C different from E C F, the course, which for every point of the rhumb-line is the same.

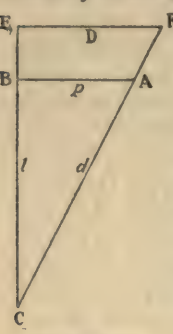


FIGURE 11

Thus a Mercator's chart presents two decided advantages for nautical purposes; viz.,

1. The vessel's track is represented by a straight line.

2. The angle which this line makes with each meridian is the course.

To find the course from one point to another on the chart, all that is necessary is to draw a line, or lay down the edge of a ruler, through the two points, and measure its angle with any meridian. A convenient mode is to refer such line by means of parallel rulers to the center of one of the compass diagrams, which usually will be found on the chart, and reading the course from the diagram.

When such diagrams are constructed with reference to the true meridian, the course obtained is the *true* course, and not the *magnetic* course.

69. The distance C F, however, is an augmented distance, which we may measure nearly by the augmented scale on the meridians of the chart (the middle latitude of the scale used being the same as that of the line C E). Or we may construct the proper distance, C A, by constructing the triangle, C B A, of Plane Sailing, in which C B is the proper difference of latitude, the scale for which is on the equator.

The distance here spoken of, though represented on this chart by a straight line, is not the shortest distance between the two points; for on the surface of a sphere, the shortest distance between two points is the arc of a great

circle which joins them. To find this belongs to *great-circle sailing*.

70. On a Polyconic Projection each parallel of latitude is developed upon its own cone, the vertex of which is on the axis at its intersection with the tangent to the meridian at the parallel. The advantage of a chart so constructed is that those portions lying near the central meridian have little or no distortion, and it also allows the use of a scale of distance anywhere on the chart. It is an excellent navigational chart, but rejected by many because its parallels are curved lines.

The method of its construction, together with tables for the same, is given in the Polyconic Projection Tables, issued by the Coast Survey.

All nations have now adopted the Mercator Projection as being the most practical for navigators. The method of construction of a projection by Mercator is as follows:

Let a projection be required for a chart of 14° extent in longitude between the parallels of latitude 20° 30′ and 30° 25′, and let the space allowable on the paper between these parallels measure 10 inches.

Entering Table 3, column headed 20°; running down to the line marked 30′ in the side column, is found 1248.9; entering the column 30°; running down to the line 25′, is found 1905.5. The difference, or 1905.5 − 1248.9 = 656.6, is the value of the meridional arc between these latitudes, for which 1′ of arc of the equator is taken as the unit. On the intended projection,

1′ of arc of longitude will measure $\frac{10^{\text{in.}}}{656.6} = 0.0152$ inch, which

will be the scale of the chart. For brevity call it 0.015. Multiply this quantity by all values derived from Table 3 before laying them down on the projection, if they are to be measured on a diagonal scale of one inch.

Draw in the center of the sheet a straight line, and assume it to be the middle meridian of the chart. Construct on this line a perpendicular near the lower border of the sheet, and assume this perpendicular to be the parallel of latitude 20° 30′; this will be the southern inner neat line of the chart. From the intersection of the lines lay off on the parallel, on each side of the middle meridian, 7° of longitude, or distances each equal to 0.015 × 60 × 7 = 6.3 inches; and through the points obtained draw lines parallel to the middle meridian, these will be the eastern and western neat lines of the chart.

In order to construct the parallel of latitude for 21° 00′, find, in Table 3, the meridional parts for 21° 00′, which are 1280.8. Subtracting from this number the number for 20° 30′, and multiplying the difference by 0.015, we obtain 0.478 inch, the distance on the chart between 20° 30′ and 21° 00′. On the meridians lay off distances equal to 0.478 inch, and through the three points obtained draw a straight line, which is the parallel of 21° 00′.

In the same manner lay down all the parallels answering to full degrees of latitude; the distances are

$$0^{in}.015 \times (1344.9 - 1248.9) = 1.440 \text{ inches.}$$
$$0^{in}.015 \times (1409.5 - 1248.9) = 2.409 \text{ inches.}$$
$$0^{in}.105 \times (1474.5 - 1248.9) = 3.384 \text{ inches, etc.}$$

This gives the parallels of latitude 22° 00′, 23° 00′, 24° 00′, etc. Lay down in the same way the parallel of latitude 30° 25′, which is the northern inner neat line of the chart.

A degree of longitude measures on this chart $0^{in}.015 \times 60 = 0^{in}.9$. Lay off, on the lowest parallel of latitude drawn on the chart, on a middle one, and on the highest parallel, measuring from the middle meridian toward each side, the distances of $0^{in}.9$, $1^{in}.8$, $2^{in}.7$, $3^{in}.6$, etc., in order to determine the points where meridians answering to full degrees cross the parallels drawn on the chart. Through the points found draw the meridians. Draw then the outer neat lines of the chart at a convenient distance outside of the inner neat lines, and extend to them the meridians and parallels. Between the inner and outer neat lines of the chart subdivide the degrees of latitude and longitude as minutely as the scale of the chart will permit; the subdivisions of the degrees of longitude are found by dividing the degrees into equal parts, and the subdivisions of the degrees of latitude are accurately found in the same manner as the full degrees of latitude previously described, though it will generally be found sufficiently exact to make even subdivisions of the degrees, as in the case of the longitude.

The subdivisions between the two eastern as well as those between the two western neat lines will serve for measuring or estimating terrestrial distances. Distances between points bearing North and South of each other are ascertained by referring them to the subdivisions between the same parallels. Distances represented by lines at an angle to the meridians (loxodromic lines) may be measured by taking between the dividers a small number of the subdivisions near the middle

latitude of the line to be measured, and stepping them off on that line. If the terrestrial length of a line running at an angle to the meridians between the parallels of latitude of 24° 0′ and 29° 00′ is required, the distance shown on the neat space between 26° 15′ and 26° 45′ (= 30 nautical miles) is taken between the dividers and stepped off on that line.

GREAT-CIRCLE SAILING

71. The *rhumb-line*, or spiral curve, which cuts all the meridians at the same angle, was used formerly by navigators in passing from point to point on account of the simplicity of the calculations required in practice. But, as has been stated, it is a longer line than the great circle between the same points, and therefore the intelligent navigators of the present day are substituting the latter wherever practicable.

On the Mercator chart, however, the arc of a great circle joining two points, not on the equator or on the same meridian, will not be projected as a straight line, but as a curve longer than the Mercator distance, and still greater than the distance on a rhumb-line. Hence it is an objection to the Mercator chart, that the shortest route from point to point *appears* on it as a circuitous one; and this is, doubtless, one main reason why merely practical men have made so little use of the great circle. Many of those unacquainted with the mathematical principles of the subject are unable to comprehend how the apparently circuitous path on their chart should actually be the line of shortest distance.

72. ***To project on a chart the arc of a great circle joining two given points on the globe.***

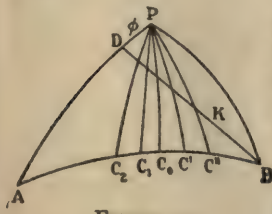


FIGURE 12

Solution. It will be necessary to project a number of points of the arc, and trace through these points the curve by hand. To project a point on the chart, we must know its latitude and longitude.

The two given points, A and B (Fig. 12), and the pole, P, are the three angular points of a spherical triangle, formed by the arcs joining these points with each other and with the pole. If from P we draw P C_0 perpendicular to A B, the point C_0 is nearer the pole than any other point of A B; that is, it is the point of maximum lati-

tude. This point of greatest latitude is called the *vertex* of the great circle.

To find the latitude and longitude of this vertex.

This may be done by a direct application of the rules of Spherical Trigonometry, first finding the angles A and B, and then solving one of the right triangles $A\,P\,C_0$ or $B\,P\,C_0$. But in practice the following method is preferable.

Let $L_1 = (90° - \text{P A})$, and λ_1 be the latitude and longitude of A, the point left.

$L_2 = (90° - \text{P B})$, and λ_2, be the same of B, the point arrived at.

$L_v = (90° - \text{P C}_0)$, and λ_v be the same of the vertex, C_0.

$\lambda = (\lambda_1 - \lambda_2)$, is the difference of longitudes of A and B.

A B $= d$, is the distance between A and B. Draw K perpendicular to P A, dividing it into P D $= \phi$ and AD $= 90° - (L_1 + \phi)$.

Then in the triangles A B D, and B D P, by Napier's Rules we have

$$\cos \lambda = \tan \phi \tan L_2 \text{ or, } \tan \phi = \cos \lambda \cot L_2 \quad (19)$$

$$\sin \phi = \cot \lambda \tan K \quad (20)$$

$$\cos (L_1 + \phi) = \cot A \tan K \text{ or,}$$

$$\cot A = \cos (L_1 + \phi) \cot \lambda \operatorname{cosec} \phi. \quad (21)$$

A is the course from A.

In the right triangle P C_0 A, we have

$$\cos L_v = \cos L_1 \sin \text{A} \quad (22)$$

$$\sin L_1 = \cot A \cot (\lambda_1 - \lambda_v) \text{ or,}$$

$$\cot (\lambda_1 - \lambda_v) = \sin L_1 \tan A. \quad (23)$$

$$\sin d \sin A = \cos L_2 \sin \lambda \text{ (check).} \quad (24)$$

73. *To find any number of points*, C′, C″, C‴, etc., C_1, C_2, C_3, etc., we may assume at pleasure the differences of longitude from the vertex C_0 P C′, C_0 P C″, C_0 P C‴, etc. It is best to assume them at equal intervals of 5° or 10°.

Let $\lambda' = C_0 \text{ P C}'$, $L' = (90° - \text{P C}')$, the lat. of C′,

$\lambda'' = C_0 \text{ P C}''$, $L'' = (90° - \text{P C}'')$, " C″,

$\lambda''' = C_0 \text{ P C}'''$, $L''' = (90° - \text{P C}''')$, " C‴

etc. etc.

then the right triangles C_0 P C′, C_0 P C″, C_0 P C‴, etc., give

$$\left.\begin{aligned} \tan L' &= \tan L_v \cos \lambda', \\ \tan L'' &= \tan L_v \cos \lambda'', \\ \tan L''' &= \tan L_v \cos \lambda''', \text{ etc.} \end{aligned}\right\} \quad (25)$$

Or we may assume values of L', L'', L''', etc., and find the corresponding values of λ', λ'', λ''', etc., by the formulas

$$\left.\begin{aligned} \cos \lambda' &= \tan L' \cot L_v, \\ \cos \lambda'' &= \tan L'' \cot L_v, \\ \cos \lambda''' &= \tan L''' \cot L_v, \text{ etc.} \end{aligned}\right\} \quad (26)$$

from which we shall have two values of λ for each value of L.

Having thus found as many points as may be deemed sufficient, we may plot them upon the chart, and through them trace the required curve.

74. ***To find the great-circle distance and course between two given points.***

Solution. Let d be the distance between the two points A and B (Fig. 12).

Then in the triangles $B\,D\,P$ and $A\,D\,B$, by Napier's Rules, we have,

$$\sin L_2 = \cos \phi \cos K, \quad (27)$$

$$\cos d = \sin (L_1 + \phi) \cos K, \quad (28)$$

$$\cos d = \sin (L_1 + \phi) \sin L_2 \sec \phi, \quad (29)$$

$$\cot d = \cos A \tan (L_1 + \phi). \quad \text{(Check.)} \quad (30)$$

d, reduced to minutes, will be the distance in geographic miles.

The course from A is found by (21).

The course from B may be found from the right triangle $B\,C_0\,P$.

$$\cos B = \sin L_v \sin (\lambda_v - \lambda_2). \quad (31)$$

The vertex lies between A and B, unless either A or B is $> 90°$.

By the cosine-haversine formula, derived in Art. 161, we have

$$\text{hav } \theta = \cos \text{L} \cos d \text{ hav } t;$$

and

$$\text{hav } z = \text{hav } (L \sim d) + \text{hav } \theta \quad (73)$$

putting z or zenith distance = distance between ports = d

$$L = L_1 \qquad d = L_2 \qquad \text{and} \qquad t = \lambda_1 - \lambda_2$$

we have,

$$\text{hav } d = \text{hav } (L_1 \sim L_2) + \cos L_1 \cos L_2 \text{ hav } (\lambda_1 - \lambda_2) \quad (32)$$

Having determined d or A B (Fig. 12) the angles A, the initial course and B, the final course may be found by the law of sines:

$$\sin A = \frac{(\sin P)(\sin P\,B)}{\sin A\,B} = \sin (\lambda_1 - \lambda_2) \cos L_2 \operatorname{cosec} d \quad (33)$$

$$\sin B = \frac{(\sin P)(\sin P\,A)}{\sin A\,B} = \sin (\lambda_1 - \lambda_2) \cos L_1 \operatorname{cosec} d \quad (34)$$

75. EXAMPLE 1: Find the great-circle distance and initial course, from San Francisco, Lat. 37° 48′ N., Lon. 122° 28′ W., to Yokohama, Lat. 35° 26′ N., Lon. 139° 39′ E. (Formulas 19, 21, 22, 23, 29).

Latitudes are both +, $\lambda_1 - \lambda_2 = 97^\circ\ 53'$.

	°	′	″									
$\lambda_1 - \lambda_2 =$	97	53		cos	−9.13722	cot	−9.14134					
$L_2 =$	35	26		cot	0.14780			sin	9.76324			
$\phi =$	169	05	21	tan	−9.28502	cosec	0.72290	sec	−0.00792			
$L_1 =$	37	48	N.							cos	9.89771	sin 9.78739
$L_2 + \phi =$	206	53	21			cos	−9.95031	sin	−9.65540			
$C =$ N.	56	52	40	W.		cot	9.81455			sin	9.92299	tan 0.18545
$d =$	74	30	44 = 4470′.75 miles					cos	9.42656			
$L_2 =$	48	33	55							cos	9.82070	
$\lambda_1 - \lambda_2 =$	46	47	26									cot 9.97284
$\lambda_1 =$	122	28	W.									
$\lambda_2 =$	169	15	26									

EXAMPLE 2: In the above example, find the coördinates of points of the curve at intervals in longitude each side of vertex of 5° up to 50° (Formula 25).

Long. from Vertex	Tan L_v	$l \cos \lambda$	$l \tan L$	Lat.	Longitudes	
					E. of vertex	W. of vertex
				° ′ ″	° ′	° ′
± 0	05421	0.00000	0.05421	48 34 N.	169 15 W.	169 15 W.*
± 5	05421	9.99834	0.05255	48 27 30	164 15	174 15
± 10	05421	9.99335	0.04756	48 08	159 15	179 15 W.
± 15	05421	9.98494	0.03915	47 35	154 15	175 45 E.
± 20	05421	9.97299	0.02720	46 48	149 15	170 45
± 25	05421	9.95728	0.01149	45 45 30	144 15	165 45
± 30	05421	9.93753	9.99174	44 27 30	139 15	160 45
± 35	05421	9.91336	9.96757	42 52	134 15	155 45
± 40	05421	9.88425	9.93846	40 57	129 15	150 45
± 45	05421	9.84949	9.90370	38 42	124 15	145 45
± 50	05421	9.80807	9.86228	36 04	119 15 W.	140 45 E.
						* (vertex)

EXAMPLE 3: In the above Example 1, find the initial course and distance (using Formulas 32, 33).

	Distance	*In-course*
$L_1 = 37° 48'$ N.	cos 9.89771	
$L_2 = 35° 26'$ N.	cos 9.91105	cos 9.91105
$\lambda_1 - \lambda_2 = 97° 53'$	hav 9.75479	sin 9.99588
	θ hav 9.56355	
	θ nat hav 36606	
$L_1 - L_2 = 2° 22'$	nat hav 00043	
$d = 74° 30' 45''$	nat hav 36649	cosec 0.01606
$= 4470.75$ miles		sin A 9.92299
		course N. 56° 52′ 40″ W. (303° 07′)

76. To follow a great circle rigorously requires a continual change of the course. It is usual to steam from point to point by compass, thus making rhumb-lines between these points.

When the vessel has deviated from the great circle which it was intended to pursue, it is necessary to make out a new one from the point reached to the place of destination. It is a waste of time to attempt to get back to an old line and a new course may be easily found from the Azimuth Tables (H. O. Pub. No. 71 and No. 120).

77. As the course, in order to follow a great circle, is practically the most important element to be determined, me-

chanical means of doing it have been devised. Towson's method and tables and Airy's method are used by English navigators.

Charts are constructed by a gnomonic projection, on which great circles are represented by straight lines; convenient means are provided on those charts for determining the course and distance without computation.

78. A great circle between two points near the equator, or near the same meridian, differs little from a rhumb-line. But when the differences both of latitude and of longitude are large, the divergence is very sensible. It is then that the great circle, as the line of shortest distance, is preferred.

But it is to be noted that in either hemisphere the great-circle route lies nearer the pole, and passes into a higher latitude, than the rhumb-line. Should it reach too high a latitude, it is usually recommended to follow it to the highest latitude to which it is prudent to go, then follow that parallel until it intersects the great circle again. This is called Composite Sailing. Great circle charts issued by the H.O. show by inspection when it is best to use the composite course.

79. A knowledge of great-circle sailing will often enable the navigator to shape his course to better advantage. Let A B (Fig. 13) be the rhumb-line on a Mercator's chart, A C B the projected arc of a great circle.

The length on the globe of the great circle A C B is less than that of the rhumb-line A B, or of any other line, as A D B, between the two. But A C B is also less than lines that may be drawn from A to B on the other side of it, that is, nearer the pole; and there will be some line, as A D′ B, nearer the pole than the great circle, and equal in length to the rhumb-line. Between this and the rhumb-line may be drawn curves from A to B, all less than the rhumb-line.

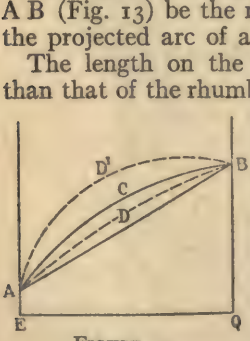


FIGURE 13

80. A striking contrast between a great circle and a rhumb would be the extreme case of a vessel from a point in a high latitude to another on the same parallel 180° distant in longitude. The great-circle course is across the pole, first north, then south while the rhumb-line is along the small circle, the parallel of latitude, east or west; the two courses differing 90°. Any arc of a small circle drawn between the two points, and lying between the pole and the parallel of latitude, will be less than the arc of the parallel.

Hence the vessel may steam on one of these small circles nearly west, and make a less distance than on the Mercator rhumb, or parallel due east. This is, indeed, an impossible case in practice, but it gives an idea of the advantage to be gained in any case by a knowledge of the great-circle route.

81. Conclusions. — Observations then applicable to all great circles are as follows:

The rhumb is always nearer the equator than the great circle.

The great circle between two places in different hemispheres, not on the same meridian, has a double curvature when laid down on a Mercator chart.

The great circle intersects every meridian at a different angle.

On the equator the great circle and rhumb coincide.

On a great circle the port of destination is always right ahead, but on the rhumb, it is only right ahead immediately before arriving there.

Economy in distance on a great-circle course over Mercator sailing is greatest in high latitudes, between places differing little in latitude.

A great-circle course can be readily found from the azimuth tables; it is only necessary to substitute for the declination, the latitude of destination, for the hour-angle, the difference of longitude between the places expressed as time, and to consider the latitude of departure as that of the observer. For marking the course, apply the rules for marking the azimuth.

CHAPTER VI

REFRACTION. — DIP OF THE HORIZON. — PARALLAX. — SEMIDIAMETERS

REFRACTION

82. It is a fundamental law of optics, that a ray of light passing from one medium into another of different density is refracted, or bent from a rectilinear course. If it passes from a lighter to a denser medium, it is bent toward the perpendicular to the surface which separates the two media; if it passes from a denser to a lighter medium, it is bent from that perpendicular. Let

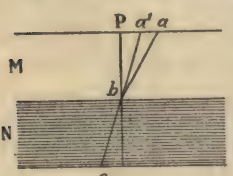


FIGURE 14

M and N (Fig. 14) represent two media, each of uniform density, but the density, or refracting power, of N being the greater;

$a\,b\,c$, the path of the ray of light through them;

P b, the normal line, or perpendicular, to the separating surface at b.

If $a\,b$ is the *incident ray*, $b\,c$ is the *refracted ray;* P $b\,a$ is the *angle of incidence;* P $b\,a'$ is the angle of *refraction.*

If $c\,b$ is the incident ray, $b\,a$ is the refracted ray, and P $b\,a'$ and P $b\,a$ are respectively the angles of incidence and refraction.

Moreover, these angles are in the same plane, which, as it passes through P b, is perpendicular to the surface at which the refraction takes place; and we have for the *refraction*

$$a'\,b\,a = \mathrm{P}\,b\,a - \mathrm{P}\,b\,a',$$

or *the difference of direction of the incident and refracted rays.*

A more complete statement of the law for the same two media is, that

$$\frac{\sin \mathrm{P}\,b\,a}{\sin \mathrm{P}\,b\,a'} = m, \text{ a constant for these media;}$$

or, *the sines of the angles of incidence and refraction are in a constant ratio.*

This law is also true when the surface is curved as well as when it is a plane.

83. If the medium N, instead of being of uniform density, is composed of parallel strata, each uniform but varying from each other, the refracted ray $b\,c$ will be a broken line; and if, as in Fig. 15, the thickness of these strata is indefinitely small, and the density gradually increases in proceeding from the surface b, $b\,c$ will become a curved line. But we shall still have for any point c of this curve, ca' being a tangent to it,

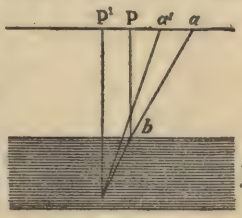


FIGURE 15

$$\frac{\sin \mathrm{P}\,b\,a}{\sin \mathrm{P}'\,c\,a'} = m,$$

a constant for the particular stratum in which c is situated.

This law, which is true for strata in parallel planes, extends also to parallel spherical strata, except that the normals P b, P' c are no longer parallel, but will meet at the center of the sphere. But the refraction takes place in the common plane of these two normals.

84. The earth's atmosphere presents such a series of parallel spherical strata, denser at the surface of the earth, and decreasing in density, until at the height of fifty miles the refracting power is inappreciable.

In Fig. 16 the concentric circles M N represent sections of these parallel strata, formed by the vertical plane passing through the star S and the zenith of an observer at A. The normals C A Z at A, and C B E at B, are in this vertical plane. S B, a ray of light from the star S, passes through the atmosphere in the curve B A, and is received by the observer at A.

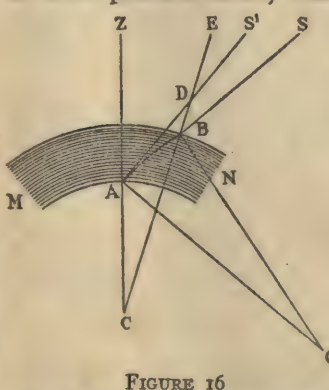


FIGURE 16

Let A S' be a tangent to this curve at A; then the *apparent* direction of the star is that of the line A S'; and the astronomical refraction is the difference of directions of the two lines B S and A S'. This difference of directions is the difference of the angles E B S, E D S', which the lines S B, S' A, make with any right line C B E,

which intersects them. If, then, r represent the refraction, we have

$$r = \text{E B S} - \text{E D S}'.$$

Also, E B S is the angle of incidence, and Z A S′, the apparent zenith distance, is the angle of refraction; and we have

$$\frac{\sin \text{E B S}}{\sin \text{Z A S}'} = m$$

a constant ratio for a given condition of the atmosphere and a given position of A; but varying with the density of the atmosphere, and for different elevations of A above the surface. For a mean state of the atmosphere and at the surface of the earth, experiments give $m = 1.000294$.

The principles of Arts. 82 and 83, applied to this case, show that astronomical refraction takes place in vertical planes, so as to increase the altitude of each star without affecting its azimuth. The refraction must therefore be subtracted from an observed altitude to reduce it to a true altitude; or

$$h = h' - r,$$

in which
h is the true altitude,
h', the apparent altitude,
r, the refraction.

These laws are here assumed. The facts and reasoning on which they depend belong to works on Optics. (Bowditch, Art. 296.)

85. After a profound investigation of the problem, Laplace obtained a complicated formula for determining the refraction. Bessel has modified and improved Laplace's formula. His tables of refraction are now considered the most reliable. They are found in a convenient form for nautical problems in Table 20, Bowditch. The mean refractions in this table are for the height of the barometer 30 inches, and the temperature 50° Fahrenheit.*

86. Tables 21 and 22, Bowditch, contain corrections to be applied to the normal refraction for changes in temperature and barometric height, deduced also from Bessel's Tables.

87. When $h = 90°$, or the object is in the zenith, $r = 0$; that is, the path is a straight line.

When $h = 0$, or the object is in the horizon, the ray of light,

* Chauvenet's Astronomy Vol. I contains a thorough investigation of the problem of refraction, especially of Bessel's formulas.

nearly horizontal, describes near the earth's surface a curve which is approximately the arc of a circle whose radius is seven times the radius of the earth; or,

$$R' = 7\,R.$$

This, however, is in a mean condition of the atmosphere. The curve is greatly varied in extraordinary states of the atmosphere, or by passing near the earth's surface of different temperatures; in very rare cases even to the extent of becoming convex to the surface a short distance.

DIP OF THE HORIZON

88. ***To find the dip of the horizon.***

Solution. Let A (Fig. 17) be the position of the observer at the height B A = h, above the level of the sea; A H, perpendicular to the vertical line, C A, represents the true horizon.

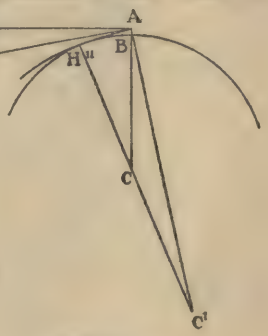


FIGURE 17

The most distant point of the horizon visible from A is that at which the visual ray, H″ A, is tangent to the earth's surface.

The apparent direction of H″ is A H′, the tangent to the curve A H″ at A. $\Delta H = H\,A\,H'$ is the dip of the horizon to be found.

Let C be the center of the earth,

C′, the center of the arc H″ A.

H″, C, C′, are in the same straight line, since the arcs H″ B, H″ A are tangent to each other at H″,

C A, C′A, are perpendicular respectively to A H, A H′; hence

$$C\,A\,C' = H\,A\,H' = \Delta H, \text{ the dip.}$$

Let R = C B, the radius of the earth;

then $R + h$ = C A,

$7\,R$ = C′ A = C′ H″, the radius of curvature of H″ A,

$6\,R$ = C C′.

We have, then, in the triangle C A C′, by plane trigonometry,

$$\sin \tfrac{1}{2}\Delta H = \sqrt{\frac{(6\,R - \tfrac{1}{2}h)\,(\tfrac{1}{2}h)}{7\,R(R+h)}};$$

and, since h is comparatively very small, and may therefore be omitted alongside of R,

$$\sin \tfrac{1}{2} \Delta \mathrm{H} = \sqrt{\frac{3\,h}{7\,R}};$$

or, putting $\sin \tfrac{1}{2} \Delta H = \tfrac{1}{2} \Delta H \sin 1''$,

$$\Delta H = \frac{2}{\sin 1''}\sqrt{\frac{3\,h}{7\,R}} = \frac{2}{\sin 1''}\sqrt{\frac{3}{7\,R}}\sqrt{h}. \qquad (35)$$

89. Taking R = 20902433 feet, we find the constant factor

$$\frac{2}{\sin 1''}\sqrt{\frac{3}{7\,R}} = 59''.071,$$

$$\Delta H = 59''.071\sqrt{h}, \qquad (36)$$

and $\log \Delta H = 1.77137 + \tfrac{1}{2} \log h$,

h being expressed in feet, which is nearly the formula for Table 14 (Bowditch).

Since $\frac{2}{\sin 1''}\sqrt{\frac{3}{7\,R}}$ is constant, depending only upon the radius of the earth, ΔH is proportional to $\sqrt{h}$, or the dip is proportional to the square root of the height of the observer above the level of the sea.

90. Were the path of the ray, H″ A, a straight line, we should have

$$\Delta' H = \mathrm{H\,A\,H''} = \mathrm{H''\,C\,A},$$

and in the triangle H″ C A,

$$\cos \Delta' H = \frac{R}{R+h},$$

whence, $2 \sin^2 \tfrac{1}{2} \Delta' H = \frac{h}{R+h} = \frac{h}{R}$, nearly,

and $$\Delta' H = \frac{2}{\sin 1''}\sqrt{\frac{h}{2\,R}},$$

or with h in feet, $$\Delta' H = 63''.803\sqrt{h}. \qquad (37)$$

Comparing this with $\Delta H = 59''.07\sqrt{h}$, we find

$$\Delta H = \Delta' H - 4''.733\sqrt{h} = \Delta' H - .074\,\Delta' H,$$

or that the dip is decreased by refraction by .074, or nearly $\frac{1}{13}$ of it.

But from the irregularity of the refraction of horizontal rays (Art. 87), the dip varies considerably, so that the tabulated dip for the height of 16 feet can be relied on ordinarily only within 2′. When the temperatures of the air and water differ greatly, variations of the dip from its mean value as great as 4′ may be experienced. In some rare cases variations of 8′ have been found.

The dip may be directly measured by a dip-sector. A series of such measurements carefully made, and under different circumstances, both as to the height of the eye, temperature and pressure of the atmosphere, and temperature of the water, is greatly needed.

91. Chauvenet (Astronomy, Vol. I) has deduced the following formula:

$$\text{in seconds, } \Delta H = \Delta' H - 24021'' \frac{t - t_0}{\Delta' H},$$

or

$$\text{in minutes, } \Delta H = \Delta' H - 6'.67 \frac{t - t_0}{\Delta' H};$$

in which t is the temperature of the air,
t_0 that of the water,

by a Fahrenheit thermometer.

When the sea is warmer than the air, the visible horizon is found to be below its mean position, or the dip is greater than the tabulated value; when the sea is colder than the air, the dip is less than its tabulated value.

This uncertainty of the dip affects to the same extent all altitudes observed with the sea horizon.

92. Near the shore, or in a harbor, the horizon may be obstructed by the land. (Bowditch, Art. 302.) The shoreline may be then used for altitudes instead of the proper horizon. Table 15 (Bowditch) contains the dip of such waterline, or of any object on the water, for different heights in feet and distances in sea miles. It is computed by the formula

$$D = \frac{3d}{7} + 0.56514 \frac{h}{d} \qquad (38)$$

in which

h is the height in feet;
d, the distance of the object in sea miles;
D, the dip in minutes.

93. ***To find the distance of an object of known height, which is just visible in the horizon.***

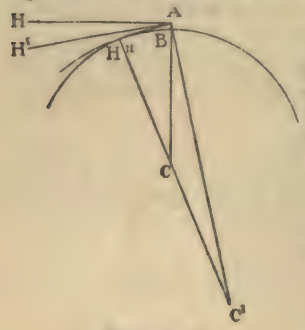


FIGURE 18

Solution. If the observer is at the surface of the earth at the point H″ (Fig. 18), a point A appears in the horizon, or is just visible, when the visual ray A H″ just touches the earth at H″. Let

h = B A, the height of A,
d = H″ A, the distance of A.

As this arc is very small, we have

$$d = \text{H}''\,\text{C}'\,\text{A} \sin 1'' \times \text{C}'\,\text{A} = 7\,R \times \text{H}''\,\text{C}'\,\text{A} \sin 1'',$$

since $\text{C}'\,\text{A} = 7\,R$.

From the three sides of the triangle C C′ A by plane trigonometry.

$$\sin \tfrac{1}{2}\,\text{H}''\,\text{C}'\,\text{A} = \sqrt{\frac{\tfrac{1}{2}\,h\,(R + \tfrac{1}{2}\,h)}{42\,R^2}},$$

or nearly $$\tfrac{1}{2}\,\text{H}''\,\text{C}'\,\text{A} \sin 1'' = \sqrt{\frac{h}{84\,R}},$$

and $$\text{H}''\,\text{C}'\,\text{A} \sin 1'' = \sqrt{\frac{h}{21\,R}}.$$

This, substituted in the expression for d, gives

$$d = 7\,R\sqrt{\left(\frac{h}{21\,R}\right)} = \sqrt{\left(\frac{7}{3}R\,h\right)}. \qquad (39)$$

In this, d, h, and R are expressed in the same denomination. But if h and R are in feet,

in statute miles, $$d = \frac{1}{5280}\sqrt{\left(\frac{7}{3}\,R\,h\right)},$$

in geographical miles, $$d = \frac{1}{6080.2}\sqrt{\left(\frac{7}{3}\,R\,h\right)}.$$

Taking R = 20902433 feet as before, we find

$$\left.\begin{aligned} &\text{in stat. miles } d = 1.323\,\sqrt{h}, \text{ or } \log d = 0.12156 + \tfrac{1}{2}\log h, \\ &\text{in geog. } \text{“ } d = 1.148\,\sqrt{h}, \text{ or } \log d = 0.05994 + \tfrac{1}{2}\log h. \end{aligned}\right\} \qquad (40)$$

The first of these is nearly the formula given in Bowditch for computing Table 6.

94. Were the visual ray, H″ A, a straight line, we should have from the right triangle *C H″ A*,

$$H'' A = \sqrt{(C A^2 - H'' C^2)}, \text{ or } d' = \sqrt{(2 R + h)}\, h;$$

or nearly $$d' = \sqrt{2 R} \times \sqrt{h}.$$

Introducing the same numerical values as before, we have in statute miles

$$d' = 1.225 \sqrt{h}.$$

Comparing this with the expression above, we see that the distance is increased about $\frac{1}{12}$ part by refraction. This, however, is subject to great uncertainty.

95. If the observer is also elevated at the height of B′ A′ (Fig. 19), and sees the object A in his horizon, then its distance is

$$A' H'' + H'' A,$$

or the sum of the distances of each from the common horizon, H″.

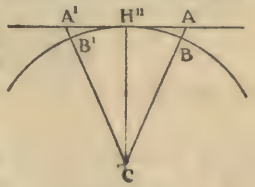


FIGURE 19

By entering Table 6 with the heights of the observer and the object respectively, the sum of the corresponding distances is the distance of the object from the observer. The distances in this table are in statute miles. Multiplying them by $\frac{5280}{6080.2} = .86839$, reduces them to geographical miles.

PARALLAX

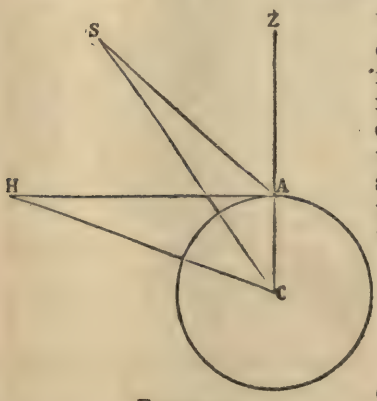


FIGURE 20

96. The change of the direction of an object, arising from a change of the point from which it is viewed, is called *parallax;* and it is always expressed by the angle at the object, which is subtended by the line joining the two points of view. Thus in Fig. 20 the object S would be seen from A in the direction A S; and from C in the direction C S. The angle at S, subtended by A C, is the difference of these directions, or the parallax for the two points of view, C and A.

97. In astronomical observations, the observer is on the surface of the earth; the conventional point to which it is

most convenient to reduce them, wherever they may be made, is the earth's center. In those problems of practical astronomy which are used by the navigator, we have only to consider this *geocentric parallax*, which is the difference of the direction of a body seen from the surface and from the center of the earth. It may also be defined to be the angle at the body subtended by that radius of the earth which passes through the place of the observer. Thus, in Fig. 20, if

C is the center of the earth, and
A the place of the observer,

the geocentric parallax of a body, S, will be the angle

$$S = Z\,A\,S - Z\,C\,S,$$

at the body subtended by the radius C A.

If the earth is regarded as a sphere, C A Z will be the vertical line through A, and will pass through the zenith, Z. Then will the plane of C A S be a vertical plane;

Z A S, the *apparent* zenith distance of S as observed at A;
Z C S, its geocentric or *true* zenith distance; and
A Z S > Z C S.

Thus we see that this parallax takes place in a vertical plane, and increases the zenith distance, or decreases the altitude, of a heavenly body without affecting its azimuth.

98. This suffices for all nautical problems, but for the more refined observations at observatories, the spheroidal form of the earth must be considered. Then, as in Fig. 21, the radius C A does not coincide with the normal or vertical line C′ A Z, but meets the celestial sphere at a point Z′, in the celestial meridian, nearer the equator than the zenith, Z.

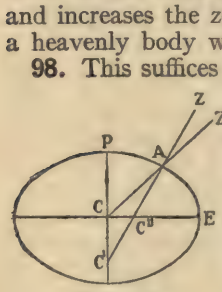


FIGURE 21

We may remark here that
A C″ E, the angle which the vertical line makes with the equator, is the latitude of A; and
A C E, the angle which the radius makes with the equator is its *geocentric* latitude.

99. ***To find the parallax of a heavenly body for a given altitude.***

Solution. In Fig. 20 let

p = S, the parallax in altitude;
z = Z A S, the apparent zenith distance of S, corrected for refraction;
R = C A, the radius of the earth;

d = C S, the distance of the body S, from the center of the earth.

Then, from the triangle C A S we have

$$\sin \text{C S A} = \frac{\text{C A}}{\text{C S}} \sin \text{C A S},$$

or,
$$\sin p = \frac{R \sin z}{d}, \tag{41}$$

If the object is in the horizon, as at H, the angle A H C is called its horizontal parallax; and denoting it by P, we have from (41), or from the right triangle C A H,

$$\sin P = \frac{R}{d}, \tag{42}$$

which, substituted in (41), gives

$$\sin p = \sin P \sin z. \tag{43}$$

If $h = 90° - z$, the apparent altitude of the object, we have —

$$\sin p = \sin P \cos h; \tag{44}$$

or nearly, since p and P are small angles,

$$\left.\begin{aligned} p &= P \cos h, \\ \text{or} \qquad p &= P \sin z. \end{aligned}\right\} \tag{45}$$

100. The horizontal parallax P is given in the Nautical Almanac for the sun, moon, and planets. From Fig. 20 it is obviously the semidiameter of the earth, as viewed from the body. As the equatorial semidiameter is larger than any other, so also will be the *equatorial horizontal parallax.* This is what is given in the Almanac for the moon. Strictly, it requires reduction for the latitude of the observer, and such reduction is made at observatories, and in the higher order of astronomical observations. It is given in Table 19 (Bowditch).

101. Tables 16 and 17 (Bowditch) are computed by formula (45).

Table 23 contains the correction of the moon's altitude for parallax and refraction corresponding to a mean value of the horizontal parallax, 57′ 30″. It should be used, however, only for very rough observations, or a coarse approximation.

102. Table 24 contains, to each minute of horizontal parallax, and every 10′ of altitude from 5°, the combined correction for parallax and refraction of the apparent altitude of the moon's center: barom., 30″; therm., 50° F. Before using this table, the observed altitude of the moon's limb should be corrected for instrumental errors, dip, and semidiameter.

APPARENT SEMIDIAMETERS

103. The apparent diameter of a body is the angle which its disk subtends at the place of the observer.

To find the apparent semidiameter of a heavenly body.

Solution. In Fig. 22 let M be the body;
d = C M, its distance from the center of the earth;

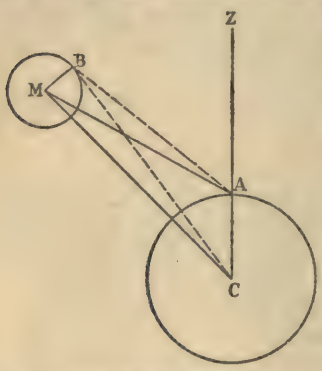


FIGURE 22

d' = A M, its distance from A;
r = M B, its linear radius or semidiameter;
s = M C B, its apparent semidiameter, as viewed from C;
s' = M A B′, its apparent semidiameter, as viewed from A (B and B′ are too near each other to be distinguished in the diagram);
R = C A, the earth's radius.

1. For finding s, the right triangle C B M gives

$$\sin s = \frac{r}{d}. \tag{46}$$

Were the body M in the horizon of A, or Z A M = 90°, its distance from A and C would be sensibly the same, so that the angle s is called the *horizontal* semidiameter.

In (42) we have for the horizontal parallax,

$$\sin P = \frac{R}{d}, \qquad \text{or } d = \frac{R}{\sin P},$$

which, substituted in (46), gives

$$\sin s = \frac{r}{R} \sin P, \tag{47}$$

or nearly, since s and P are small,

$$s = \frac{r}{R} P, \tag{48}$$

$\frac{r}{R}$ is constant for any particular body, as it is simply the ratio of its linear diameter to that of the earth.

For the moon,

$$\frac{r}{R} = 0.272,$$

$$\left.\begin{aligned} s &= 0.272\ P, \\ \text{and} \qquad \log s &= 9.43457 + \log P. \end{aligned}\right\} \tag{49}$$

By this formula the moon's horizontal semidiameter may be found from its horizontal parallax. (The American Ephemeris and Nautical Almanac, 1918, page 719.)

The Nautical Almanac contains the semidiameters as well as the horizontal parallaxes of the sun, moon, and planets.

2. For finding s′, the apparent semidiameter as viewed by an observer at A on the surface of the earth, the right triangle A B′ M gives

$$\sin s' = \frac{r}{d'}. \tag{50}$$

In the triangle C M A,

$$\frac{\sin \text{M A C}}{\sin \text{M C A}} = \frac{\text{C M}}{\text{A M}},$$

or, putting $h = 90° - \text{Z A M}$, the apparent,
and $h' = 90° - \text{Z C M}$, the true altitude of M,

$$\frac{\cos h}{\cos h'} = \frac{d}{d'}, \tag{51}$$

whence,

$$d' = d\,\frac{\cos h'}{\cos h},$$

which, substituted in (50), and by (46), gives

$$\sin s' = \frac{r\ \cos\ h}{d \cos h'} = \sin s \frac{\cos h}{\cos h'},$$

or approximately,

$$s' = s\,\frac{\cos h}{\cos h'}, \tag{52}$$

by which s' may be found when s and h are known.

Since $h < h'$, $\cos h > \cos h'$, and consequently $s' > s$; that is, the semidiameter increases with the altitude of the body. The excess

$$\Delta s = s' - s, \text{ is called the augmentation.}$$

The moon is the only body for which this augmentation is sensible. It is given in Table 18 (Bowditch).

CHAPTER VII

TIME

104. Transit. — The instant when any point of the celestial sphere is on a given meridian is designated as the transit of the point over that meridian.

105. Hour-angle. — The hour-angle of any point of the sphere is the angle at the pole which the circle of declination passing through the point makes with the meridian. It is properly reckoned from the upper branch of the meridian, and positively toward the west. It is usually expressed in hours, minutes, and seconds of time. The intercepted arc of the equator is the measure of this angle.

106. Sidereal Time. — The intervals between the successive transits of any fixed point of the sphere (as, for instance, of a star which has no proper motion) over the same meridian would be perfectly equal, were it not for the *variable* effect of nutation. This correction, arising from a change in the position of the earth's axis, is most perceptible in its effect upon the transit of stars near the vanishing point of that axis, i.e., near the poles of the heavens. Hence, for the exact measurement of time, we use the transits of some point of the equator, as the *vernal equinox*. This point is often called the *first point of Aries*. Its usual symbol is ♈.

107. The interval between two successive transits of the vernal equinox is a *sidereal day;* and such a day is regarded as commencing at the instant of the transit of that point. The sidereal time is then $0^h\,0^m\,0^s$. This instant is sometimes called *sidereal noon*.

The effect of nutation and precession in changing the time of the transit of the vernal equinox is so nearly the same at two successive transits, that the sidereal days thus defined are sensibly equal. It is unnecessary, then, except in refined discussions, to discriminate between *mean* and *apparent* sidereal time.

108. The *sidereal time* at any instant is the hour-angle of the vernal equinox at that instant, and is reckoned on the equator from the meridian westward around the entire circle;

that is, from 0 to 24^h. It is equal to the right ascension of the meridian at the same instant.

109. Solar Time. — The interval between two successive transits of the sun over a given meridian is a *solar day*, and the hour-angle of the sun at any instant is the *solar time* of that instant.

In consequence of the motion of the earth about the sun from west to east, the sun appears to have a like motion among the stars at such a rate that it increases its right ascension daily nearly 1°, or 4^m of time. With reference to the fixed stars, it therefore arrives at the meridian each day about 4^m later than on the previous day; consequently solar days are about 4^m longer than sidereal days.

110. Apparent and Mean Solar Time. — If the sun changed its right ascension uniformly each day, solar days would be exactly equal. But the sun's motion in right ascension is not uniform, varying from $3^m\ 35^s$ to $4^m\ 26^s$ in a solar day. There are two reasons for this, —

1. The sun does not move in the equator, but in the ecliptic.

2. Its motion in the ecliptic is not uniform, being most rapid at the time of the earth's perihelion, about January 1, and slowest at the time of the aphelion, about July 2.

To obtain a uniform measure of time depending on the sun's motion, the following method is adopted. A fictitious sun, called a *mean sun*, is supposed to move uniformly in the *ecliptic* at such a rate as to return to the perihelion and aphelion at the same time with the true sun. A *second mean sun* also supposed to move uniformly in the *equator* at the same rate that the first moves in the ecliptic, and to return to each equinox at the same time with the first mean sun.

The time which is measured by the motion of this second mean sun is uniform in its increase, and is called *mean* time.

That which is denoted by the true sun is called *true* or *apparent* time.

The difference between mean and apparent time is called the *equation of time*. It is also the difference of the right ascensions of the true and mean suns.

The instant of transit of the true sun over a given meridian is called *apparent noon*. The instant of transit of the second mean sun is called *mean noon*. The mean time is then $0^h\ 0^m\ 0^s$.

Mean noon occurs, then, sometimes before and sometimes after apparent noon, the greatest difference being about 16^m, early in November.

111. Astronomical Time. — The solar day (apparent or mean) is regarded by astronomers as commencing at noon (apparent or mean), and is divided into 24 hours, numbered successively from 0 to 24.

Astronomical time (apparent or mean), is then, the hour-angle of the sun (true or mean) reckoned on the equator *westward* throughout the entire circle from 0^h to 24^h.

112. Civil Time. — For the common purposes of life, it is more convenient to begin the day at midnight; that is, when the sun is on the meridian below the horizon, or at the sun's lower transit. The civil day begins 12^h before the astronomical day of the same date; and is divided into two periods of 12^h each, namely, from midnight to noon, marked A.M. (ante-meridian), and from noon to midnight, marked P.M. (post-meridian). Both apparent and mean time are used.

The affixes A.M. and P.M. distinguish civil time from astronomical time. During the P.M. period, this is the only distinction, — the day, hours, etc., being the same in both.

113. To convert civil into astronomical time, it is only necessary to drop the A.M. or P.M., and when the civil time is A.M., deduct 1^d from the day, and increase the hours by 12^h.

To convert astronomical into civil time, if the hours are less than 12^h, simply affix P.M.; if the hours are 12^h or more than 12^h, deduct 12^h, add 1^d, and affix A.M.

When the civil time is between 12^h and 1^h as $12^h\ 10^m$ P.M. Jan. 4, 1918, the astronomical time is Jan. 4, $0^h\ 10^m$.

When the civil time is $12^h\ 10^m$ A.M., Jan 4, 1918, the astronomical time is $12^h\ 10^m$ Jan. 3.

ASTRONOMICAL TIME.						CIVIL TIME.					
	d	*h*	*m*	*s*			*d*	*h*	*m*	*s*	
1918 May	10	14	15	10	= 1918 May		11	2	15	10	A.M.
1918 Sept.	8	9	19	20	= 1918 Sept.		8	9	19	20	P.M.

114. The hour-angle of the sun (true or mean), at any meridian, is called the *local* (apparent or mean) solar time. The hour-angle of the sun (true or mean) at Greenwich at the same instant is the corresponding *Greenwich* time.

So also the hour-angle of ♈ at any meridian, and its hour-angle at Greenwich at the same instant, are corresponding *local* and *Greenwich* sidereal times.

115. ***The difference of the local times of any two meridians is equal to the difference of longitude of those meridians.***

Demonstration. In Fig. 23, let

P M, P M′ be the celestial meridians of two places;

P S, the declination circle through the sun (true or mean);

M P S, the hour-angle of the sun at all places whose meridian is P M, will be the local time (apparent or mean) at those places; so also

M′ P S will be the corresponding local time at all places whose meridian is P M′; and

M P M′ = M P S − M′ P S will be the difference of longitude of the two meridians.

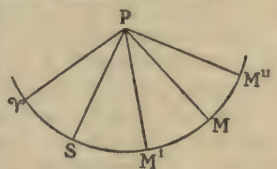


FIGURE 23

If P ♈ is the equinoctial colure,

M P ♈ and M′ P ♈ will be the corresponding sidereal times at the two meridians; still, however,

M P M′ = M P ♈ − M′ P ♈.

The proposition is true, then, whether the times compared are *apparent*, *mean*, or *sidereal*.

The difference of longitude is here expressed in time. It is readily reduced to arc by observing that

$$\left.\begin{aligned} 24^h &= 360^\circ \\ 1^h &= 15^\circ \\ 1^m &= 15' \\ 1^s &= 15'' \end{aligned}\right\} \quad \text{or} \quad \left\{\begin{aligned} 1^\circ &= 4^m \\ 1' &= 4^s \\ 1'' &= \tfrac{1}{15} \end{aligned}\right.$$

In comparing corresponding times of different meridians, the most easterly meridian is that at which the time is *greatest*. For converting arc into time and the reverse by inspection, see Haversine Tables (Bowditch Tables, 45).

116. If (Fig. 23) P M is the meridian of Greenwich,

M P S is the Greenwich solar time, and

M P M′ the longitude of the meridian P M′.

M P M′ = M P S − M′ P S;

so also P M M′ = M P ♈ − M′ P ♈;

or, *the longitude of any meridian is equal to the difference between the local time of that meridian and the corresponding Greenwich time.*

117. If we put

T_0 = M P S, the Greenwich time,

T = M′ P S, the corresponding local time,

λ = M P M′, the longitude of the meridian, P M′,

we have $\lambda = T_0 - T,$

and $T_0 = T + \lambda,$ (53)

in which λ is + for west longitudes, and T_0 and T are supposed to be reckoned always *westward* from their respective meridians from 0^h to 24^h; that is, T_0 and T are the *astronomical* times, which should always be used in all astronomical computations.

118. Usually the *first* operation in most computations of nautical astronomy is to convert the local mean time into the corresponding astronomical time (Art. 113).

The Greenwich time should never be otherwise expressed than astronomically. On this account it would be convenient to have chronometers intended for nautical or astronomical purposes marked from 0^h to 24^h, instead of 0^h to 12^h as is now the customary practice.

119. The *second* operation often required is to convert the local astronomical time into Greenwich time. For this we have (53), which numerically is

$$T_0 = T \pm \lambda \left\{ \begin{array}{l} + \text{ when the longitude is } \textit{west}, \\ - \text{ when it is } \textit{east}, \end{array} \right.$$

and, in words, gives the following

Rule. Having expressed the local time astronomically, *add* the longitude, if *west; subtract* it, if *east;* the result is the corresponding Greenwich time, or a rule easily remembered,

Longitude west, Greenwich time best.
Longitude east, Greenwich time least.

TIME

EXAMPLE 1: April 1, 1918, in Lon. 76° 32′ W., the local time is $9^h\ 3^m\ 10^s$ A.M.; what is the Greenwich time?

Local Ast. T. =	March	31^d	21^h	$3^m\ 10^s$
Longitude =		+	5	6 8
G. T. =	April	1	2	9 18

EXAMPLE 2: March 20, 1918, in Lon. 30° E., the local time is $6^h\ 3^m$ A.M., what is the G. T.?

Loc. Ast. T. =	March	19^d	18^h	3^m
Long. =		−	2	0
G. T. =	March	19	16	3

EXAMPLE 3: August 21, 1918, in Lon. 105° 15′ E., the local time is $4^h\ 3^m$ P.M., what is the G. T.?

Loc. Ast. T. =	August	21^d	4^h	3^m
Long. =		−	7	1
G. T. =	August	20	21	2

By reversing this process, that is, by *subtracting* the longitude if *west*, or adding it if *east*, we may reduce the Greenwich time to the corresponding local time.

When observations are noted by a chronometer regulated to Greenwich time, an *approximate* knowledge of the longitude and local time is necessary in order to determine whether the chronometer time is A.M., or P.M., and thus fix the true Greenwich date. If the time is A.M., the hours must be increased by 12^h.

EXAMPLE 1: August 3, 1918, in Lon. 5^h W., about 3^h P.M., the Greenwich chronometer read $8^h\ 11^m\ 7^s$, and is fast of G. T. $6^m\ 10^s$. What is the Greenwich time?

Approx. Loc. T.	Aug. $3^d\ 3^h$	G. Chro.	$8^h\ 11^m\ 7^s$
Lon.	$+\ 5$	Correction	$-\ 6\ 10$
Approx. G. T.	Aug. $3^d\ 8^h$	G. T. Aug.	$3^d\ 8^h\ 4^m\ 57^s$

EXAMPLE 2: December 7, 1918, in Lon. 10^h E., about 1^h A.M., the G. Chro. read $3^h\ 14^m\ 13^s.5$, and is fast $25^m\ 18^s.7$. Find the G. T.

Approx. Loc. T. Dec.	$6^d\ 13^h$	G. Chro.	$3^h\ 14^m\ 13^s.5$
Long.	$-\ 10$	Correction	$-\ 25\ 18^s.7$
Approx. G. T. Dec.	$6^d\ 3^h$	G. T. Dec.	$6^d\ 2^h\ 48^m\ 54^s.8$

EXAMPLE 3: February 13, 1918, in Lon. $9^h\ 12^m$ W., about 2^h A.M., the G. Chro. read $11^h\ 27^m\ 13^s.3$, and is fast $30^m\ 30^s.3$. Find the G. T.

Approx. Loc. T. Feb.	$12^d\ 14^h\ 0^m$	G. Chro.	$11^h\ 27^m\ 13^s.3$
Long.	$+\ 9\ 12$	Correction	$-\ 30\ 30^s.3$
Approx. G. T. Feb.	$12^d\ 23^h\ 12^m$	G. T. Feb.	$12^d\ 22^h\ 56^m\ 43^s.0$

The operations on the approximate times may be performed mentally.

120. Standard Time. — By this system, introduced for the convenience of railways and now adopted by the commercial world, the civil mean time of certain standard meridians is used throughout the adjacent districts. The standard meridians are one hour (15°) apart, and those used in the United States and Canada are the 60th, 75th, 90th, 105th and 120th meridians west from Greenwich; the times are designated respectively Intercolonial, Eastern, Central, Mountain, and Pacific. In Alaska the 135th meridian is used. The belts of territory for $7\frac{1}{2}°$ on each side of a standard use as far as practicable the time of that meridian.

To reduce Local Mean Time to Standard Time. — If the

local meridian is E. of the standard, subtract the difference of longitude between the two meridians from the l. m. t., and if W., add it. (Table 36, Bowditch.)

121. Change in Vessel's Clocks due to Change of Longitude. — It is very convenient at sea to keep the vessel's clocks set so as to show local mean time as nearly as possible.

When a vessel whose clocks keep 120th meridian time steams from San Francisco toward Honolulu, on arriving at the 135th meridian the longitude is changed 15° W., or one hour in time to the westward so that the time at this meridian is one hour earlier; now, to keep the time somewhere near the local time of the vessel's position, it becomes necessary to put the clocks back. Steaming from Honolulu to San Francisco, for every 15° of longitude to the eastward, the time is one hour later, so the clocks must be set ahead. The sun moves westward during the day, so when a vessel steams westward, it takes the sun more than 24 hours by the clock to return to the vessel's meridian, so that each day at sea will be longer than 24 clock hours by an amount equal to the change in the vessel's longitude expressed in time. When steaming eastward, each day at sea will be shorter than 24 clock hours; therefore, when steaming east put the clocks ahead; when steaming west put the clocks back.

It is the custom on most vessels to change the clocks during the night or in the morning watch, so that the change will interfere little with the vessels work and also show correct time at noon, so that the time will be known when to take the meridian altitude for latitude.

EXAMPLE 1: September 21, 1918, on a passage from San Francisco (Lat. 37° 37′ N., Lon. 123° W.) to Honolulu, a vessel steams S. 62° W., 16 knots. Find how many minutes a day the clocks must be set back.

In 24 hours the vessel steams $24 \times 16 = 384$ miles.

Course	dist.	d. lat	dep.	d. lo
S. 62° W.	384	180.3	339	419

Lat. 37° 37′ 00″ N.
l 3 00 18 S.
Lat. in 34 36 42 N.

419′ = 6° 59′ W. = 28^m of time

The clocks should be put back 28^m a day of 24 clock hours.

The vessel day is now 28^m longer than 24 clock hours. Run in 28^m = 7.5 miles S. 62° W = diff. long. of 8′ W. or 32^s. Therefore the clocks should be put back $28^m\ 32^s$ or about 29^m a day.

122. Let us now consider the change in time when we arrive at the meridian of 180° W. on a passage between Honolulu and Yokohama. When in 180° W. the vessel's clocks will be 12 hours behind G. M. T. at the same instant of time.

EXAMPLE 1: December 7, 1918, at $4^h\ 00^m\ 00^s$ P.M. a vessel arrives in Lat. 25° N. Lon. 179° 50′ W., and at $5^h\ 00^m\ 00^s$ P.M. the vessel is in 179° 50′ E. Find the date at the last position.

	h	m	s
L. M. T. Dec. 7	4	00	00
Lon. W.	11	59	20
G. M. T. Dec. 7	15	59	20
1 hour later	+1	00	00
G. M. T. Dec. 7	16	59	20
Lon. E.	11	59	20
L. M. T. Dec. 8	4	58	40

The date therefore goes one day ahead at 5 : 00 P.M.

EXAMPLE 2: July 4, 1918, a vessel on a passage from Yokohama to Honolulu arrives at 179° 50′ E., at $9^h.00$ A.M. and at 10^h : 00 A.M. is in Lon. 179° 50′ W. Find the date at the last position.

	h	m	s
L. M. T. July 3	21	00	00
Lon. E.	11	59	20
G. M. T. July 3	9	00	40
1 hour later	+1	00	00
G. M. T. July 3	10	00	40
Lon. W.	11	59	20
L. M. T. July 2	22	01	20
or July 3	10^h	01^m	20^s A.M.

The date therefore goes back a day at 10 A.M.

The possibility of using a wrong Greenwich date when navigating about the 180° Meridian can be avoided by noting that the sequence of the days of the month used in the Greenwich date from day to day remain unbroken.

CHAPTER VIII

THE NAUTICAL ALMANAC*

123. The American Ephemeris and Nautical Almanac is divided into three parts as follows: Part I, Ephemeris for the meridian of Greenwich, gives the ephemerides of the sun and moon, the geocentric and heliocentric positions of the major planets, the sun's coördinates, and other fundamental astronomical data for equidistant intervals of Greenwich mean time; Part II, Ephemeris for the meridian of Washington, gives the ephemerides of the fixed stars, sun, moon, and major planets for transit over the meridian of Washington, and Part III, Phenomena, contains predictions of phenomena to be observed with data for their computation. Tables are also appended for the interconversion of mean and sidereal time and for finding the latitude and azimuth by an altitude of Polaris.

The American Nautical Almanac is a smaller book made up of extracts from the "Ephemeris and Almanac" just described, and is designed especially for the use of navigators, being adapted to the meridian of Greenwich. The elements dependent upon the sun and moon are placed in the first part of the book, arranged according to hours, days, and months of the year. The right ascension of the mean sun for the entire year is given at one opening, also, the mean time of sidereal noon at Greenwich. Solar and lunar eclipses are described, and tables for the interconversion of mean and sidereal time and for finding the latitude by Polaris are included.

124. The Planets. — The apparent right ascension and the apparent declination of the four planets, Venus, Mars, Jupiter, and Saturn, are given for every Greenwich mean noon throughout the year, referred to the true equator and equinox. The time of transit, meridian of Greenwich, is given to the nearest minute. The apparent right ascensions and the apparent

* See extracts from Ephemeris and Almanac for 1918. Appendix I.

declinations are accompanied by the difference or change in every 24-hour interval. By means of these differences interpolations may be conveniently made to any Greenwich mean time by proportional parts (Table IV), using the difference as the argument at the top of the page and the G. M. T. as the argument at the right-hand side of the page. The time of transit meridian of Greenwich can be interpolated by simple inspection to any other meridian, by interpolating forward for west longitudes and backward for east longitudes. The semidiameter and horizontal parallax of the planets are given at the foot of the pages for the first day of each month.

The right ascension and declination of a planet are required whenever it is observed for time, latitude, or azimuth.

125. Stars. — The right ascension, and declination, in a list of apparent places of 55 stars, are given for the time of meridian passage at Greenwich on the first day of every month, also the star's historical name and its magnitude.

For the same 55 stars the G. M. T. of transit at Greenwich are given on the first day of every month, also the correction for reducing the data from the first to any other day of the month.

The mean places, with their annual variations, of 110 additional stars are also given for the beginning of the current year.

The right ascension of a star is the sidereal time of its meridian passage. The mean time of meridian passage may, therefore, be roughly found from the right ascension by adding the mean time of sidereal noon at Greenwich.

The right ascension and declination of a star are required whenever it is observed for time, latitude, or azimuth.

126. The Sun. — The sun's declination and the equation of time are given for every even hour throughout the year; interpolation may be made by inspection and is facilitated by the H. D. (Hourly Difference) which is given at the end of each day. At the end of the month is given the Sun's semidiameter for every tenth day. The Sun's declination is required whenever that body is observed for latitude, local time, or azimuth. The equation of time is the apparent time of Greenwich mean noon, or the hour-angle of the true Sun at that instant. When interpolated to any given G. M. T., it is the correction to be applied to mean time in order to obtain apparent time. The Sun's semidiameter is used in reducing the

altitude of the upper or lower limb of the Sun to the altitude of the center; and in reducing the angular distance between the limb of the Sun and any other object to the distance from the center of the Sun.

The Sun's right ascension and declination, given in the Almanac, are referred to the true equator and equinox; are corrected for aberration, and are, therefore, apparent positions.

127. Signs. — It is to be noted that the positive sign used with declinations indicates north and the negative sign south.

The sign + must be used with the H. D. when the equation of time or Sun's declination, if itself positive, is increasing, or if negative, is decreasing numerically; contrariwise, the sign − must be used with the H. D. when the equation of time or Sun's declination, if positive, is decreasing, or if negative, is increasing numerically.

128. The Moon. — The Moon's right ascension and declination are given every even hour throughout the year, referred to the true equator and equinox, and also the Moon's semidiameter and horizontal parallax. The right ascension and declination are accompanied by the difference or change in every two-hour interval; by means of these differences, interpolation may be conveniently made to any G. M. T. by proportional parts, (Table IV,) using the difference in two hours as the argument at the top of the page, and the interval from the nearest even hour of G. M. T. as the argument at the left-hand side of the page. The semidiameter and horizontal parallax may be taken directly from the Almanac without interpolation; they are required for all observations of the Moon.

The phases of the Moon are given for the entire year; these are likewise to be found, as they occur, at the foot of the pages under the right ascensions and declinations.

The Moon's mean time of transit, meridian of Greenwich is given accompanied by the difference in minutes between the times of day of successive transits. The local time of the Moon's meridian passage at any given place may be found from that at Greenwich by means of these differences, together with the longitude of the place measured from Greenwich and expressed in time, and proportional parts (Table IV), using the differences between the times of successive Greenwich transits as the argument at the top of the page, and the longi-

tude of the given place from Greenwich as the argument at the right-hand side of the page, interpolating forward for west longitudes and backward for east longitudes.

129. ***Greenwich time is essential in taking from the almanac the various elements.***

130. The quantities in the Nautical Almanac or in the Ephemeris, as commonly in other mathematical tables, are approximate numbers, that is, each is given only to the nearest unit of the lowest retained order; and no refinement of interpolation can give a result to a higher degree of precision. In interpolating, more than one lower order in any case is superfluous. Thus, the sun's declination is given to the nearest $0''.1$, in the Ephemeris, and in no way can we by interpolation obtain a value which will be reliable within a narrower limit.

Moreover, the Greenwich times are uncertain to a greater or less extent; and if *first* differences only are used, the interpolated result can be regarded as true only within much wider limits than the approximation of the Almanac or of the Ephemeris.

In interpolating, then, it is well to consider the degree of approximation which is wanted in any particular case; and if the nearest $1'$, or $10''$, or $1''$ suffices, contract the interpolation so as to retain at the most one lower order; or else, consider the degree of approximation attainable in any particular case, and contract the work so as to retain only the reliable figures. All lower orders are superfluous, and are deceptive, as giving the appearance of a higher degree of accuracy than has actually been obtained; as, for instance, using *tenths* and *hundredths* of seconds, when the data will give a result reliable within $2'$ or $3'$ only.

131. Should it be desirable to interpolate more accurately than can be done by first differences alone, the reduction for *second* differences may be introduced by a simple process.

Let Δ_2 be the change of Δ_1 in the time T'. Then, instead of Δ_1, as found in the Ephemeris and Nautical Almanac for the nearest Greenwich date, we may substitute

$$\Delta_1 + \frac{t}{2\,T'}\Delta_2; \qquad (54)$$

that is, the value of Δ_1, interpolated for $\frac{1}{2}\,t$, or to the middle instant between the Almanac date and the given time. This is simply using the mean rate of change during the interval.

If Δ_1 is a "Diff. for 1^h" given for the Almanac for each day, $T' = 24^h$; if Δ_1 is a "Diff. for 1^m" given in the Almanac for each hour, $T' = 60^m$.

The interpolation of Δ_1 to the middle instant may often be performed mentally.

EXAMPLE: January 30, 1918, the hourly difference of the sun's right ascension at Greenwich mean noon was $10^s.272$. At the same time on January 31, it was $10^s.238$. Find the mean hourly rate at G. M. T. $8^h\ 30^m$.

For Jan. 30 0^h $\Delta_1 = 10^s.272$
" Jan. 31 0^h $\Delta_1 = 10^s.238$
$\Delta_2 = -0^s.034$

By interpolation for Jan. 30, $4^h.25$ the middle instant between Jan. 30, 0^h and Jan. 30, $8^h\ 30^m$.

$$\frac{4.25}{24} \text{ of } 0^s.034 = 0^s.006$$

$$\Delta_1 = 10^s.272 - 0^s.006 = 10^s.266$$

which is the mean hourly change in the interval from 0^h to $8^h\ 30^m$.

132. In the following examples the required quantities are taken from the Nautical Almanac, and interpolated to the nearest second by first differences, which suffices for the practical purposes of navigation.

EXAMPLE 1: For a L. M. T., July 13, 1918, $10^h\ 03^m\ 30^s$ A.M. in Lon. 85° 15′ W., find the sun's declination, equation of time, right ascension of the mean sun, and semidiameter; also the sun's apparent right ascension.

Find first the G. M. T.

	h	m	s	
Local astronomical mean time July 12	22	3	30	
Longitude from Greenwich W.....	5	41	0	
Greenwich mean time July 13.....	3	44	30	$= 3^h.74$

To find the sun's declination

	Sun's Declination °	′
July 13, 2^h G. M. T.....	+21	54.5
H. D. $-0'.4 \times 1.74$.....		−0.7
	N. 21	53.8

To find the equation of time

	Equation of Time m	s
July 13, 2^h G. M. T.....	−5	28.4
H. D. $-0^s.3 \times 1.74$		−0.5
	−5	28.9

(− to mean time)

To find semidiameter

July 11, S. D. = 15′.76
July 21, S. D. = 15 .77
therefore, S. D. July 13, $3^h.74$ = 15′ 46″.

To find right ascension of mean sun

	R. A. M. S.
July 13, at Greenwich mean noon	$7^h\ 22^m\ 13^s.6$
Corr. $3^h\ 44^m\ 30^s$ (Tab. III)*	0 36 .9
R. A. M. S.	7 22 50 .5

To find sun's right ascension

	h	m	s
R. A. M. S. at G. M. N. July 13	7	22	13.6
Correction for $3^h\ 44^m\ 30^s$		+0	36.9
Eq. of Time (sign changed)		+5	28.9
Apparent R. A.	7	28	19.4

EXAMPLE 2: Find the right ascension, declination, semidiameter, and horizontal parallax of the moon on January 25, 1918, at 6 P.M. in Lat. 30° N., Lon. 77° 30′ W.

Local astronomical mean time Jan. 25	$6^h\ 00^m\ 00^s$
Longitude from Greenwich W.	5 10 00
Greenwich mean time Jan. 25	$11^h\ 10^m\ 00$
	$= 11^h.17$

	Moon's R. A.	H. D.
Jan. 25, 10^h	7 31 08	12^{9s}
Corr.	+ 2 30.9	$1^h.17$
Jan. 25, $11^h\ 10^m$ R. A. =	7 33 38.9	Corr. $+150^s.93$

	Moon's Dec.	H. D.
Jan. 25, 10^h	+19° 59′.4	−7′.5
Corr.	−8 .8	$1^h.17$
Jan. 25, $11^h\ 10^m$ Dec. =	N. 19 50.6	Corr. −8′.8

(By proportional parts Table IV, Nautical Almanac)

The nearest even hour is 12; the interval is 50^m; the difference or increase of right ascension in 2^h is 257; the difference of declination in 2^h is 152 and is southward, or negative. With the arguments 50^m and 257 take out the change in right ascension; and with the arguments 50^m and 152 take out the change in declination. Subtract the changes, since the interpolation is here made backward.

	Right Ascension			Declination	
	h	m	s	°	′
January 25, $12^h\ 0^m$	7	35	26	+19	44.3
Change in 50^m		1	47	−0	6.3
January 25, $11^h\ 10^m$	7	33	39	N. 19	50.6

* Or Tab. 9, Bowditch.

The Moon's Semidiameter and Horizontal Parallax are found by inspection and S. D. = 15′.1; H. P. = 55′.4.

EXAMPLE 3: Find the right ascension, and declination of Venus, January 5, 1918, at 7 P.M. in Lat. 30° N., Lon. 60° W.

G. M. T. = Jan. 5, $11^h\ 0^m\ 0^s$.

	Venus's R. A.	*H. D.*
Jan. 5, at Greenwich mean noon	$21^h\ 46^m\ 06^s$	$+4^s.17$
Corr.	+ 45.8	G. M. T. = 11^h
Jan. 5, 11^h	R. A. = 21 46 51.8	Corr. = +45′.8

	Venus's Dec.	*H. D.*
Jan. 5, at G. M. noon	−12° 41′.7	+8′.33
Corr.	+ 9′.2	G. M. T. = 11^h
Jan. 5, 11^h	Dec. = S. 12° 32′.5	corr. +9′.16

EXAMPLE 4: Find the sun's equation of time, and declination for local apparent noon on December 3, 1918, at sea in Lat. 35° 00′ N., Lon. 64° 30′ W.

When the longitude is west the G. A. T. of local noon is equal to the longitude, for we have

	h	*m*	*s*
Local ast. app. time Dec. 3	0	0	0
Lon. W.	4	18	
G. ast. app. t. Dec. 3	4	18	0

It is therefore afternoon of the same date by the number of hours of longitude, but in East longitude at local app. noon the G. A. T. is before noon of the local date by the number of hours of longitude.

	h	*m*	*s*
Local ast. app. time Dec. 3	0	0	0
Lon. E.	−4	18	
G. A. T. Dec. 3	−4	18	
or Dec. 2	19	42	

then for 64° 30′ W. the

G. A. T. of local apparent noon	$= 4^h\ 18^m\ 00^s$
Eq. of T.	− 10 16
G. M. T. of local apparent noon	= 4 07 44
	$= 4^h.13$

	m	*s*	*H.D.*
Eq. of t. 4^h	= 10	16.3	− 1.0
Corr.	−	0.3	$\lambda = 0^h.3$
Eq. t.	= 10	16	Corr. $-0^s.3$

(+ to mean time)

		H.D.
Dec. 4^h	$-22° 03'.5$	$-0'.4$
Corr.	$-\ 0\ .1$	G. M. T. $= 0^h.13$
Dec. at L. A. noon	$= S.\ 22° 03'.6$	Corr. $-.052$

133. ***To find the mean time of the moon's transit over a given meridian on a given day.***

Solution. The Almanac contains the mean time of each transit of the moon over the meridian of Greenwich. This mean time is the hour-angle of the mean sun (Art. 114) when the moon is on the meridian; and is therefore the difference of right ascension of the moon and the mean sun. As this difference is constantly increasing, in consequence of the moon's more rapid increase of right ascension, the mean time of each transit is later than that of the one preceding by a number of minutes, varying, according to the rate of the moon's motion from 40^m to 66^m.

If, then, T_1 and T_2 denote the mean times of two successive transits of the moon over the Greenwich meridian, $T_2 - T_1$ is the *retardation* of the moon in passing over 24^h of longitude; so that for any longitude λ (expressed in hours) the retardation is nearly

$$\frac{\lambda}{24}(T_2 - T_1). \qquad (55)$$

The mean time of a transit is, then, reduced from the Greenwich to any other meridian by interpolating for the longitude; *forward*, if the longitude is *west; backward*, if the longitude is *east*, since east longitudes are regarded as negative.

The American Ephemeris gives also the hourly differences, which facilitate the interpolation. For greater exactness, these differences may be interpolated for *half* the longitude. The practical rule will be:

Take from the Almanac the mean time of meridian passage for the given *astronomical* * day, and *add* to it the product of the "Diff. for 1^h" by the longitude in hours, if the longitude is *west; subtract* that product if the longitude is *east;* or it may be taken from Table 11 (Bowditch). The mean time of meridian passage for the given day, and that for the day *following* in *west* longitude, or for the day *preceding* in *east* longitude are those which are commonly used. But it is more exact

* It is important to notice whether the mean time of transit is more or less than 12^h. In the former case, the astronomical day is 1^d less than the civil day.

to use half the difference of the times of meridian passage for the day preceding and the day following the given day: $\frac{1}{24}$ of this is the "Diff. for 1^h." The Ephemeris now contain the times of upper and lower transit.

The times of transit are given only to tenths of a minute, which suffices the purposes of the navigator. They may be found more exactly for any meridian by the method hereafter given in Art. 144.

134. ***To find on a given day the mean time of transit of a planet over a given meridian.***

Solution. The mean time of each meridian passage at Greenwich is given, in the Almanac, for each planet. It may be reduced to any meridian in the same way as for the moon; except that, in the case of an *acceleration*, the sign of the reduction is reversed.

EXAMPLE: In Lon. 100° 15′ W., find the time of the meridian transit of the moon and Jupiter, January 25, 1918 (civil date), then the corresponding G. M. T.

Long. 100° 15′ W. = $6^h\ 41^m$ (Table 45, Bowditch)

Moon	*h*	*m*
G. M. T. of Greenwich transit, Jan. 25.	11	16
Corr. for Long. West	+	13.4
L. M. T. of local transit, Jan. 25	11	29.4
Lon. W.	+6	41
G. M. T. of local transit, Jan. 25	18	10.4

Retardation
For 1^h, 2^m
$\lambda = +6^h.7$
Corr. $+13^m.4$
or (Tab. 11, Bowd.)
$+13^m.1$

Jupiter	*h*	*m*
At Greenwich noon, Jan. 25	7	41
Corr. Lon. W.		−1.1
L. M. T. of local transit, Jan. 25	7	39.9
Lon. W.	6	41
G. M. T. of local transit, Jan. 25.	14^h	$20^m.9$

Difference
$24^h = -4^m.0$
$6^h.7 = -1.1$

NOTE. — Considering acceleration (−) Lon. W. (+) and retardation (+) Lon. E., (−) the rule of signs determines what the sign of corr. is.

In Lon. 100° 15′ E. find the time of the meridian transit of the moon and Jupiter, January 21, 1918 (civil date), then the G. M. T. of local transit.

Lon. 100° 15′ E. = $6^h\ 41^m$ E. (Table 45, Bowditch)

Moon	*h*	*m*
G. M. T. of Gr. transit, Jan. 21	7	42
Corr. Lon. E.	−	15.6
L. M. T. of local transit, Jan. 21	7	26.4
Lon. E.	−6	41
G. M. T. of local transit, Jan. 21	0	45.4

Retardation

$1^h = 2^m.33$

$\lambda = -6^h.7$

Corr. $-15^m.6$

(Tab. II, Bowd.)

-16^m

Jupiter	*h*	*m*
At Greenwich noon, Jan. 21	7	57
Corr. Lon. E.	+	1.1
L. M. T. of local transit, Jan. 21	7	58.1
Lon. E.	6	41
G. M. T. of local transit, Jan. 21	0	17.1

Difference

h	*m*
24	= −4.0
6.7	= −1.1

135. ***To find the right ascension or declination of the moon, or a planet, at the time of its transit over a given meridian on a given day.***

Solution. Find the local mean time of transit, as in Art. 133; deduce the corresponding Greenwich time by applying the longitude; and for this Greenwich time of local transit take out the right ascension or declination as in Art. 132.

If the time of transit has been noted by a clock or chronometer, regulated to either local or Greenwich time, it should be used in preference to the time of transit computed from the Almanac.

136. ***To find the sun's declination and equation of time at local apparent noon using 2d differences*** —

The American Ephemeris and Nautical Almanac is used to determine this problem.

Since we must obtain the declination, and equation of time for apparent noon, it is accomplished by reducing the local apparent time to Washington apparent time, instead of to Greenwich time. The ephemerides of the sun for Washington apparent noon are given on pages 514–521.

The most common use of this problem is to find the declination of the sun, and equation of time, in equal altitudes for determining on shore the chronometer error.

On page 684 of the Ephemeris are given the latitudes and longitudes of the principal observatories of the world.

EXAMPLE 1: April 15, 1918, at Sydney, Australia, Lat. 33° 51′ 41″ S., Lon. 151° 12′ 20″ E. find the sun's dec-

lination and equation of time at local apparent noon (2d differences).

λ Washington = $5^h\ 08^m\ 15^s.8$ W. (Page 684.)
λ_1 Sydney = $10\ 04\ 49.3$ E. = $151^\circ\ 12'\ 20''$ E.
$\lambda + \lambda_1 = 15\ 13\ 05.1$ = Lon. east of Washington = $-15^h.218$

Sun's declination

	°	′	″
Washington app. noon,			
April 15.....	+9	39	00.4
Corr.....	−	13	39.44
Dec. at local app. noon			
Apr. 15.....	N. 9	25	20.96

Eq. of Time

	m	*s*
Wash. app. noon, April 15..	+00	08.17
Corr.....	+	9.47
Eq. of T. at local app. noon,		
April 15.....	+0	17.64
(+to apparent time)		

H. D. (Wash. app. noon); change. (Dec.)

	″
H. D. Apr. 14...	+54.12
H. D. Apr. 15...	+53.72
Change, 24^h.....	−0.40
Change, 1^h.....	−0.1667
Change $\frac{\lambda + \lambda_1}{2}$,...	+0.127
H. D. Apr. 15...	+53.72
H. D. (Mean)...	+53.847
$\lambda + \lambda_1$=.....	$-15^h.218$
Corr.....	−819″.444
	$= -13'39''.444$

H. D. (W. A. noon); change (Eq. of T.)

H. D. Apr. 14.....	$-0^s.631$
H. D. Apr. 15.....	−0.618
Change, 24^h.....	+0.013
Change $\frac{\lambda + \lambda_1}{2}$.....	−0.004
H. D. Apr. 15.....	−0.618
H. D. (Mean).....	−0.622
$\lambda + \lambda_1$.....	$-15^h.218$
Corr.....	$+9^s.466$

To find the dec. and eq. of t. at local app. midnight proceed as in the above problem, using for W. A. T. (12^h − (λ_1 of place + λ of Wash.) = $12^h - 15^h.218 = -3^h.218$ Apr. 15, 1918. In West Lon. use ($12^h + \lambda_1$ of place − λ of Wash.)

CHAPTER IX

CONVERSION OF THE SEVERAL KINDS OF TIME.— RELATION OF TIME AND HOUR-ANGLES

CONVERSION OF TIME

137. ***To convert apparent into mean time, or mean into apparent time.***

Solution. For the same instant, let

T_m represent the local mean time;

T_a, the local apparent time; and

E, the equation of time with the sign of its application to *apparent* time.

Then, since the equation of time is the difference of mean and apparent times (Art. 110),

$$\left.\begin{aligned} T_m &= T_a + E, \\ T_a &= T_m - E. \end{aligned}\right\} \qquad (56)$$

The reduction, then, is made by finding from the Almanac the equation of time for a given mean time, and applying it to the given time according to the sign as given at the foot of the page.

138. ***To convert a mean into a sidereal time interval, or a sidereal into a mean time interval.***

Solution. The sidereal year is 365.25636 mean solar days, or 366.25636 sidereal days; so that the same interval of time which is measured by $365^d.25636$ reckoned in *mean* time, is measured by $366^d.25636$ if reckoned in *sidereal* time. Since both are uniform measures of time, if we represent any interval by

t, if expressed in *mean* time,

s, if expressed in *sidereal* time, then

$$\frac{s}{t} = \frac{366.25636}{365.25636} = 1.0027379;$$

whence

$$s = 1.0027379\, t = t + .0027379\, t, \qquad (57)$$

$$t = 0.9972696\, s = s - .0027304\, s, \qquad (58)$$

by which the reduction from one to the other may be made.

The computation is facilitated by Table II * of the Almanac or the Ephemeris, for converting *sidereal into mean solar time*, which contains for each second of s the value of .0027304 s; and by Table III,* for converting *mean solar into sidereal time*, which contains for each second of t the value of .0027379 t.

Tables 8 and 9 (Bowditch) contain the same quantities.

139. If in (57) $t = 24^h$; $s = 24^h\ 3^m\ 56^s.5553$; or in a *mean* solar day sidereal time *gains* on mean time $3^m\ 56^s.5553$. In 1^h of mean time the gain is $9^s.8565$.

If in (58) $s = 24^h$; $t = 24^h - 3^m\ 55^s.9094$; or in a *sidereal* day mean time *loses* on sidereal time $3^m\ 55^s.9094$. In 1^h of sidereal time the loss is $9^s.8296$.

If t and s in the last term are expressed in *hours* (57), and (58) become

$$\left.\begin{aligned} s &= t + 9^s.8565\ t \\ t &= s - 9^s.8296\ s; \end{aligned}\right\} \qquad (59)$$

by which the reductions may be more readily calculated, when the tables are not at hand.

140. To convert mean time at a given place into sidereal time.

Solution. Let

λ represent the longitude of the place, expressed in time, + when *west*,
T, the local *mean* time,
S, the corresponding *sidereal* time,
t, the interval from mean noon in *mean* time (differing from T only by omitting the day),
s, the same interval in *sidereal* time,
S_0, the sidereal time of mean noon at Greenwich,
S_0', the sidereal time of mean noon at the place;
then, since λ expresses the Greenwich time of local noon,

$$\left.\begin{aligned} &S_0' = S_0 + .0027379\ \lambda; \\ \text{evidently} \quad &S = s + S_0' \\ \text{and by (57)} \quad &s = t + .0027379\ t; \end{aligned}\right\} \qquad (60)$$

whence we have

$$S = t + S_0 + .0027379\ (\lambda + t). \qquad (61)$$

The Almanac contains S_0 for each Greenwich mean noon, under the head "Right Ascension of the Mean Sun." It should

* Tables II and III of the Almanac are not given in the extracts in Appendix I, so the student must use Tab. 8 and 9, Bowditch.

be taken out for the given astronomical day of the place; .0027379 λ is then the reduction for longitude, *additive* in *west* longitude, *subtractive* in *east*. It, as well as .0027379 t, the reduction to a sidereal interval, may be taken from Table III of the Almanac, or from Table 9 (Bowditch); or either may be computed by (57), or first of (59).

From (61), then, we have the following rule:

To the local mean time add the sidereal time of Greenwich mean noon of the given astronomical day, the reduction of this sidereal time for the longitude of the place, and the reduction of the hours, minutes, etc., of the mean time to a sidereal interval.

The astronomical (solar) day is usually retained. But if it be desirable to state the sidereal day, as well as the hours, etc., of the sidereal time, we prefix to S_0 the sidereal day at the instant of mean noon, which is the same as the astronomical day after the vernal equinox of each year; one day less before that date. At the instant of the vernal equinox the sidereal time and mean solar time coincide. Before that time the mean sun transits before the vernal equinox; after that time it transits after the vernal equinox.

141. $T + \lambda$ is the Greenwich mean time. When this is given, or found in the course of computation, it will be more convenient to take out S_0 for the Greenwich day, and the combined reduction, .0027379 $(t + \lambda)$, for the hours, minutes, etc., of Greenwich mean time, instead of for t and λ separately.

It should be noted, however, that in the first method (Art. 140), S_0 is taken out for the local day; in this, it is taken out for the Greenwich day, provided $\lambda + t$, as used, expresses properly the Greenwich time.

142. $S_0 + .0027379\ (t + \lambda)$ is the "sidereal time" of the Almanac interpolated for the Greenwich mean time. It is more convenient to term it the *right ascension of the mean sun:* and then the translation of (61) will be, *the sidereal time is equal to the right ascension of the mean sun + the mean time.*

This is also evident from Fig. 24, in which

P is the pole;
P M, the meridian;
Υ, the vernal equinox;
Υ M, the equator.
Υ M is also the *right ascension of the meridian*, and measures
M P Υ, the hour-angle of Υ, or the *sidereal* time (Art. 108).

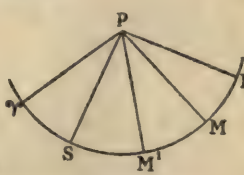


FIGURE 24

If P S is the declination-circle passing through the mean sun, ♈ S is the right ascension of the mean sun, and

M P S is its hour-angle or the *mean* time (Art. 114), and is measured by the arc of the equator, S M.

Evidently $$\Upsilon M = \Upsilon S + S M. \qquad (62)$$

The hour-angles M P ♈, M P S, are reckoned from the meridian toward the west; hour-angles east from the meridian are then regarded as negative.

If P S is the declination-circle of the true sun, then will

♈ S be the right ascension, and
M P S the hour-angle of the *true* sun; and
S M will measure the *apparent* time,

and the interpretation of (62) will be, *the sidereal time is equal to the right ascension of the true sun + the apparent time.*

EXAMPLE 1: January 30, 1918, in Lat. 0°, Lon. 150° 13′ 10″ W. the local mean time is $10^h\ 15^m\ 26^s.6$ P.M. Find the local sidereal time.

$\lambda = 150°\ 13'\ 10''$ W. $= 10^h\ 00^m\ 52^s.7$ (Tab 45, Bowditch).

	h	m	s
Local ast. mean time Jan. 30	10	15	26.6
Lon. W.	+10	00	52.7
G. M. T., Jan. 30	20	16	19.3

	h	m	s
R. A. M. S. (S_0), Jan. 30	20	35	38.6
Reduction G. M. T. (Tab. III)		3	19.8
Add local ast. mean time	10	15	26.6
Local sidereal time (rejecting 24^h)	6	54	25.0

EXAMPLE 2: September 21, 1918, at sea in Lat. 30° 25′ N., Lon. 160° 12′ 00″ E., the L. M. T. is $4^h\ 40^m\ 30^s$ A.M. Find the L. S. T.

First solution

	h	m	s
Local ast. mean time, Sept. 20	16	40	30
Lon. E.	−10	40	48
G. M. T., Sept. 20	5	59	42

	h	m	s
R. A. M. S. (S_0), Sept. 20	11	54	15.9
Red. G. M. T. (Tab. III)	+	0	59.0
Add local ast. mean time	16	40	30.0
L. S. T. (rejecting 24^h)	4	35	44.9

Second solution	*h*	*m*	*s*
Local ast. M. T. Sept. 30	16	40	30
Long. E.	−10	40	48
G. M. T. Sept. 20	5	59	42
R. A. M. S.	11	54	15.9
Red. (Tab. III) for G. M. T.		0	59.0
Greenwich sidereal time	17	54	56.9
Lon. E.	+10	40	48.0
Local sidereal time (rej. 24^h)	4	35	44.9

To find the local sidereal time from the local apparent time apply the longitude to the apparent time, this gives G. A. T.; apply the eq. of time, this gives G. M. T.: then to this G. M. T. apply the longitude, which gives the L. M. T., then proceed as in the first example.

143. ***To convert sidereal time at any place into mean time.***

1st Solution. The sidereal time at mean noon at the place is from (60)

$$S_0' = S_0 + .0027379\ \lambda;$$

the sidereal interval from mean noon,

$$s = S - S_0' = S - S_0 - .0027379\ \lambda; \qquad (63)$$

and from (58) the corresponding mean time interval,

$$t = s - .0027304\ s. \qquad (64)$$

The mean time T is completed by prefixing to t the astronomical day.

From (63) and (64) we have the following rule:

From the local sidereal time subtract the sidereal time of Greenwich mean noon of the given astronomical day and the reduction of this sidereal time for the longitude of the place; and from the sidereal interval thus obtained subtract the reduction to a mean time interval; and to the result prefix the given astronomical day.

The local sidereal time may be increased by 24^h if necessary. The reduction for longitude, .0027379 λ, may be taken from Table III of the Almanac, or from Table 9 (Bowditch); *numerically*, it is *subtractive* in *west* longitude, *additive* in *east*, as applied to the given sidereal time. The reduction of the sidereal interval, .0027304 *s*, may be taken from Table II, or from Table 8 (Bowditch), and is always subtractive.

2d Solution. Let

M_0 represent the "mean time of the *preceding* sidereal 0^h" at Greenwich;

M_0', the "mean time of the *preceding* sidereal 0^h" at the place;
S, the interval from 0^h in *sidereal* time;
t, the same interval in *mean* time:

then, since λ will be the sideral interval between the Greenwich and local sidereal 0^h,

$$M_0' = M_0 - .0027304\,\lambda,$$

evidently, $$T = t + M_0',$$

and by (58) $$t = S - .0027304\,S;$$

whence we have

$$T = S + M_0 - .0027304\,(\lambda + S). \qquad (65)$$

The Nautical Almanac contains, M_0 for the Greenwich sidereal 0^h on each *mean* day. The Almanac date of the *preceding* sidereal 0^h is generally the *same* as the local astronomical date when the sidereal time is *less* than the "sidereal time at mean noon," but 1^d *less* when the sidereal time is *greater* than that at mean noon. The doubtful case is when the mean time is within 4^m of noon: the comparison must then be made with the sidereal time at the nearest local mean noon.

The reduction of M_0 to the local meridian is $-.0027304\,\lambda$, which may be taken from Table II, or from Table 8 (Bowditch). It is *subtractive* in *west* longitude, *additive* in *east*.

The reduction of the sidereal interval, $.0027304\,S$, may be taken from the same tables; it is always *subtractive*.

The combined reduction, $.0027304\,(\lambda + S)$, may be taken out for the Greenwich sidereal time, $(\lambda + S)$, instead of for λ and S separately; but with these precautions, that when $\lambda + S > 24^h$, M_0 may be taken out for 1^d later than stated in the previous precept, and interpolated for the excess of $(\lambda + S)$ over 24^h; and when $(\lambda + S)$ is negative, to retain its negative character, or else take out M_0 for one day earlier.

3d Solution. From (61) we have

$$t = S - [S_0 + .0027379\,(t + \lambda),] \qquad (66)$$

so that, when the Greenwich mean time $(t + \lambda)$ is sufficiently known, we may find for it the right ascension of the mean sun (Art. 142),

$$S_0 + .0027379\,(t + \lambda),$$

and subtract it from the given sidereal time: or, *the mean time is equal to the sidereal time — the right ascension of the mean sun.* So also we have from Art. 142 the precept: *the apparent time is equal to the sidereal time — the right ascension of the true sun.*

EXAMPLE 1: January 30, 1918, at sea, in Lat. 0°., Lon. 150° 13′ 10″ W., the local sidereal time is $6^h\ 54^m\ 25^s$. Find the local mean time.

The astronomical day is Jan. 30; the longitude in time, = $10^h\ 00^m\ 52^s.7$ W.

First solution	*h*	*m*	*s*
R. A. M. S. at G. M. N., Jan. 30	20	35	38.6
Reduction for Lon. W. (Tab. III)		+1	38.7
Sidereal time, or R. A. M. S. at local mean noon, Jan. 30	20	37	17.3
The given sidereal time ($+24^h$, if necessary for the following subtraction)	30	54	25.0
Subtracting the first from the second gives the sidereal interval from noon	10	17	07.7
Reduction for $10^h\ 17^m\ 07^s.7$ (Tab. II)		−1	41.1
Required astronomical mean time, Jan. 30, P.M.	10	15	26.6

Second solution	*h*	*m*	*s*
Mean time at Greenwich sidereal noon, Jan. 30	3	23	48.0
Reduction for longitude W. (Tab. II)		−1	38.4
Mean time of *preceding* local sidereal noon, Jan. 30	3	22	09.6
Add the given sidereal time	6	54	25.0
Reduction for $6^h\ 54^m\ 25^s.0$ (Tab. II)		−1	07.9
Required astronomical mean time, Jan. 30	10	15	26.7

If there is any doubt about the mean time of the *preceding* local sidereal noon, the first solution is to be preferred; in fact the first solution is the one used by navigators.

EXAMPLE 2: September 21, 1918, A.M. at sea in Lat. 30° 25′ N., Lon. 160° 12′ 00″ E., the local sidereal time is $4^h\ 35^m\ 44^s.9$. Find the L. M. T.

Ast. date is Sept. 20. Lon. = $10^h\ 40^m\ 48^s$ E.

	h	*m*	*s*
R. A. M. S. at G. M. N. Sept. 20	11	54	15.9
Red. Lon. E. (Tab. III)	−	1	45.3
Sidereal time at L. M. N., Sept. 20	11	52	30.6
Given sidereal time ($+24^h$)	28	35	44.9
Sidereal int. from noon	16	43	14.3
Red. sidereal int. (Tab. II)		−2	44.3
Req. ast. mean time, Sept. 20	16	40	30.0

or Sept. 21, $4^h\ 40^m\ 30^s$ A.M.

EXAMPLE 3: January 10, 1918, at Philadelphia, in Lon. 75° 16′ 42″ W., when the noon signal (75th meridian) was received, a sidereal chronometer read $19^h\ 13^m\ 26^s.5$. Later it was compared with a mean chronometer and the sidereal chronometer then read $19^h\ 50^m\ 37^s.0$ and the mean chronometer

read $5^h\ 39^m\ 08^s.0$. Find the error of the sidereal chronometer on local sidereal time, also the mean chronometer on Greenwich mean time. (Use the Ephemeris.)

	h	m	s
Mean noon (75th mer. time)	0	00	00
Lon. 75° W.	+5	00	00
G. M. T. (75th mer.) noon, Jan. 10	5	00	00
R. A. M. S. (S_0) Jan. 10	19	16	47.44
Corr. G. M. T. (Tab. III)	+		49.282
G. S. T. (75th mer.) noon	0	17	36.722
Lon. (Philadelphia)	5	01	06.81
L. S. T. (75th mer.) noon	19	16	29.912
Chron. (sidereal) reading (75th mer.) noon	19	13	26.5
Chron. (sidereal) *error* on L. S. T. slow	0	03	03.412
Chron. (sidereal) comparison with mean chro.	19	50	37.0
L. S. T. at instant of the comparison	19	53	40.412

	h	m	s
R. A. M. S., Jan. 10	19	16	47.44
Red. Lon. ($5^h\ 01^m\ 06^s.8$), Tab. III.	+	0	49.465
Sidereal time, local 0^h	19	17	36.905
L. S. T. inst. of comparison	19	53	40.412
Sidereal int. from noon	0	36	03.507
Red. sidereal to mean time interval (Tab. II)	−	0	05.908
L. M. T. at inst. of comparison	0	35	57.599
Lon. (Philadelphia)	+5	01	06.81
G. M. T. inst. of comp.	5	37	04.409
Chro. (mean) at comp.	5	39	08.0
Chro. (mean) *error* on G. M. T., fast		2	03.591

RELATION OF TIME AND HOUR-ANGLES

144. ***To find the mean time of meridian transit of a celestial body, the longitude of the place or the Greenwich time being known.***

Solution. In the case of the sun the instant of meridian transit is *apparent noon* of the place; for which we have (56)

$$T_m = E, \text{ the equation of time,}$$

which can be taken and interpolated for the Greenwich mean time. When E is subtractive, the subtraction from the number of days can be performed.

The apparent right ascension of any body at the instant of its meridian transit is also the right ascension of the meridian, or *sidereal time.* (Art. 108.) It suffices therefore to find the right ascension of the body, and, regarding it as the *sidereal* time, reduce it to *mean* time by Art. 143.

The right ascensions of the moon are given for every two hours, and of the planets for each noon, of Greenwich mean time, and may be found for a given Greenwich mean time. If, however, the longitude of the place is given, the local mean time of transit of the moon, or a planet, may first be found from the Almanac to the nearest minute (Arts. 133, 134); then for this mean time the right ascensions of the moon, or of the planet, and of the mean sun, may be computed. Subtracting the right ascension of the mean sun from the right ascension of the moon or planet, will give the mean time of transit (Art. 143, 3d Solution). If it differ sensibly from that previously obtained, the process may be repeated with this new approximation.

If the time of transit has been noted by a chronometer, regulated either to local or Greenwich time, it should be used in preference to the approximate time of transit found from the Almanac in computing the right ascensions.

This solution will give the time of the upper culmination of a heavenly body. To find the time of a lower culmination, 12^h may be added to the right ascension of the body, if sufficiently well known; or, as is generally preferable, 12^h may be added to the longitude of the place. The instant of a lower culmination on any meridian will be that of an upper culmination on the opposite meridian.

145. ***To find the hour-angle of the sun for a given place and time.***

Solution. The hour-angle of the sun, reckoned from the upper meridian toward the west, is the *apparent* time reckoned astronomically (Art. 114). Its hour-angle east of the meridian is negative, and numerically equal to 24^h — the apparent time.

A given *mean* or *sidereal* time must then be converted into *apparent* time; for this, the longitude, or the Greenwich time, must be known approximately.

146. ***To find the hour-angle of the moon, a planet, or a fixed star, for a given place and time.***

Solution. In Fig. 25, as described in Art. 139,

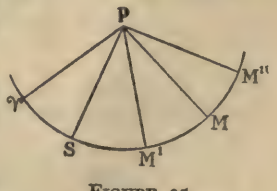


FIGURE 25

Υ M is the right ascension of the meridian, and measures M P Υ, the *sidereal* time.

Let

P S be the declination-circle of the *mean* sun, then

Υ S is the right ascension of the mean sun, and

M P S is the *mean* time, and is measured by the arc of the equator, S M.

Let

P M′ be the declination-circle of some other celestial body; then

Υ M′ is its right ascension, and

M P M′ is its hour-angle, and is measured by the arc M′ M.

From the figure,

$$M' M = \Upsilon M - \Upsilon M' = \Upsilon S + S M - \Upsilon M'. \quad (67)$$

If Υ S is the right ascension of the *true* sun,

S M will measure the *apparent* time.

From (67), then, we have the following rule:

To a given *mean* time add the right ascension of the *mean* sun, to find the corresponding *sidereal* time. Then from the *sidereal* time subtract the body's right ascension; the difference is the hour-angle west from the meridian. If it is more than 12^h, it may be subtracted from 24^h: the hour-angle, then, is —, or east of the meridian. It is necessary to know the longitude, or the Greenwich time, sufficiently near to find the right ascensions of the sun and body.

147. ***To find the local time, given the hour-angle of the sun and the Greenwich time.***

Solution. The hour-angle reckoned westward is itself the local *apparent* time, which may be reduced to *mean* or *sidereal* time (Art. 137 and 138), as may be required. The Greenwich time, or the longitude of the place, is needed only for this reduction.

148. ***To find the local time, given the hour-angle of some celestial body and the Greenwich time.***

Solution. Find from the Almanac for the Greenwich time the right ascension of the body. Then, from (67), we have

$$\Upsilon M = \Upsilon M' + M' M,$$

from which, and Arts. 140, 142, we have the following rule, regarding hour-angles to the east as negative:

To the right ascension of the body add its hour-angle; the result is the *sidereal* time. From this subtracting the right ascension of the *mean* sun gives the *mean* time.

The Greenwich time is needed for finding the required right ascensions.

Should the longitude of the place be given, but not the Greenwich time, we may first use an estimated Greenwich time, found by looking in the almanac for the time the body transits the meridian at the place, then finding the approximate G. M. T. by knowing the value of the hour angle east or west of the meridian as the case may be, and then revise the computations with a corrected value, until the assumed and computed values sufficiently agree. But the Greenwich time can be given in most practical problems.

149. ***Examples.***

EXAMPLE 1: January 27, 1918, in Lat. 40° N., Lon. 150° 13′ 15″ W., find the hour angles of the Moon and Jupiter at $8^h\ 59^m\ 45^s$ P.M., L. M. T.

	h	m	s
L. M. T., Jan. 27....	8	59	45
Lon. W.....	+10	00	53
G. M. T., Jan. 27....	19	00	38
	or $19^h.0$		

	h	m	s
L. M. T., Jan. 27...	8	59	45
R. A. M. S. (S_0)....	20	23	48.9
Red. G. M. T. (Tab. III)		3	07.4
L. S. T.....	5	26	41.3

R. A. and H. A. of the moon

	h	m	s
☽'s R. A. 18^h.....	9	24	30
Corr. 1^h.....	+	1	54.5
☽'s R. A.....	9	26	24.5
L. S. T.....	5	26	41.3
☽'s H. A.....	= −3	59	43.2
	or East $3^h\ 59^m\ 43^s.2$		

H. D. $+ 114^s.5$
G. M. T. 1^h
$+ 114^s.5$
Corr. $= 1^m\ 54^s.5$

R. A. and H. A. of Jupiter

	h	m	s
R. A. Jupiter, Jan. 27.....	3	57	59
Corr.....		+	1.5
R. A. Jupiter.....	3	58	00.5
L. S. T.....	5	26	41.3
H. A. of Jupiter.....	+1	28	40.8
	or West $1^h\ 28^m\ 40^s.8$		

H. D. $+ 0^s.08$
G. M. T. $= 19^h$
Corr. $+ 1^s.5$

An inspection of the almanac shows that the moon transits the meridian of Greenwich on January 27, 1918, at $12^h\ 49^m$ P.M.

and that Jupiter transits the same meridian on the same date at $7^h\ 33^m$ P.M. therefore at about $9^h\ 0^m$ P.M. at the vessel it is about 3^h + east of meridian for the moon and about $1^h\ 30^m$ west of the meridian for Jupiter. This gives a hasty check on the work.

EXAMPLE 2: At sea, April 8, 1918, in Lat. by D. R. 39° 48′ N., Lon. 55° 13′ W. at $7^h\ 15^m\ 20^s$ P.M. by deck watch, took an observation of Polaris to find the hour angle and the local sidereal time. A comparison of the watch, with the chronometer which was slow $10^m\ 30^s$, showed the C–W to be $3^h\ 20^m\ 15^s$.

	h	m	s
Watch	7	15	20
C – W	3	20	15
C	10	35	35
C. C.	+	10	30
G. M. T., Apr. 8	10	46	05

	h	m	s
R. A. M. S.	1	03	44.2
Red. G. M. T. (Tab. 9 Bowd.)		1	46.1
G. M. T.	10	46	05.0
G. S. T.	11	51	35.3
Lon. W.	3	40	52.0
L. S. T.	8	10	43.3
R. A. Polaris	1	29	58.2
H. A. Polaris	+6	40	45.1

or West $6^h\ 40^m\ 45^s$

Polaris

	h	m	s
R. A., Apr. 1	= 1	29	56.8
R. A., May 1	= 1	29	62.2
Diff. 30^d			+5.4

$$\text{Diff. } 8^d = \frac{8}{30} \times +5^s.4 = \frac{41.2}{30} = +1^s.37$$

	h	m	s
R. A. Polaris, Apr. 1	= 1	29	56.8
Change 8 days			+ 1.4
R. A. Polaris, Apr. 8	1	29	58.2

An inspection of the almanac shows that Polaris transits the upper meridian of Greenwich on April 8, at $0^h\ 25^m$.

Observations of Polaris to determine latitude (Art. 212) should be made when practicable near the times of upper or of lower transit (hour angle 0^h or 12^h). However, at sea, if made near elongation (hour angle 6^h or 18^h), as in the above example, the hour angle, and hence the local time, should be known within one minute.

EXAMPLE 3: January 27, 1918, G. M. T. $19^h\ 00^m\ 00^s$ the

Moon's H. A. is $3^h\ 59^m\ 43^s.2$ east of the meridian of a certain place; find the local mean time.

	h	m	s
R. A. M. S., Jan. 27	20	23	48.9
Corr. G. M. T. (Tab. III)		3	07.4
R. A. M. S.	20	26	56.3

R. A. of moon	h	m	s
Jan. 27, 18^h	9	24	30
Corr. 1^h		+1	54.5
Corr. R. A. ☽	9	26	24.5
☽'s H. A. East	−3	59	43.2
L. S. T.	5	26	41.3
R. A. M. S.	20	26	56.3
L. M. T., Jan. 27	8	59	45.0

H. D. $114^s.5$
G. M. T. 1^h
Corr. + $114^s.5$

CHAPTER X

NAUTICAL ASTRONOMY

ALTITUDES. — AZIMUTHS. — HOUR-ANGLES AND TIME

150. NAUTICAL ASTRONOMY comprises those problems of Spherical Astronomy which are used in determining geographical positions, or in finding the corrections of the instruments employed. In general they admit of a much more refined application on shore, where more delicate and stable instruments can be used, than is possible at sea, where the instability of the waves and the uncertainty of the sea-horizon present practical obstacles, both to precision in observations and to the accuracy of the results, which cannot be obviated.

151. In the problems which are here discussed, the following notation will be employed:

L = the latitude of the place of observation;
h = the true altitude of a celestial body;
$z = 90° - h$, its zenith distance;
d = its declination;
p = its polar distance;
t = its hour angle;
Z = its azimuth.

Let the diagram (Fig. 26) represent the projection of the celestial sphere on the plane of the horizon of a place:

Z, the zenith of the place; N Z S, its meridian;
P, the elevated pole, or that whose name is the same as that of the latitude;
M, the position of a celestial body;
Z M H, a vertical circle; and
P M, a declination-circle, through M.

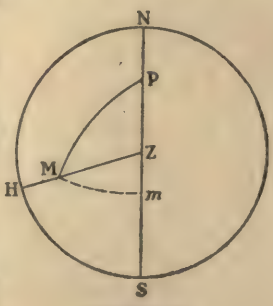


FIGURE 26

Then, in the spherical triangle P M Z,
P Z = $90° - L$, the co-latitude of the place;
P M = $p = 90° - d$, the polar distance of M;

Z M $= 90° - h$, the complement of its altitude or its zenith distance;
Z P M $= t$, its hour-angle;
P Z M $= Z$, its azimuth.

The angle P M Z is rarely used, but is sometimes called the *position angle* of the body.

This triangle, from its involving so many of the quantities which enter into astronomical problems, is called the *astronomical triangle.* As three of its parts are sufficient to determine the rest, if three of the five quantities L, d, h, t, and Z are known, the other two may be found by the usual formulas of spherical trigonometry. These admit, however, of modifications which better adapt them for practical use. The following articles point out how L, d, h, and t may be obtained.

152. The *latitudes* and *longitudes* of places on shore are given upon charts, but more accurately in tables of geographical positions, such as are found in books of sailing directions, and in Appendix IV (Bowditch). At sea it is sometimes necessary to assume them from the dead reckoning brought forward from preceding, or carried back from subsequent, determinations. (Bowditch, Art. 202.)

153. The *altitude* of an object may be directly measured at sea above the sea-horizon with a sextant; on shore, with a sextant and artificial horizon, or with a transit. All measurements with instruments require correction for the errors of the instrument. Observed altitudes require reduction for refraction and parallax; for semidiameter, when a limb of the object is observed; and at sea, for the dip of the horizon. The reductions for dip and refraction are *subtractive;* for parallax, *additive.* Strictly, the reductions should be made in the following order: for *instrumental errors*, *dip*, *refraction*, *parallax*, *semidiameter.* In ordinary nautical practice it is unnecessary to observe this order.

Following it we should have,

1. The reading of the instrument with which an altitude is measured;
2. The corrected reading or *observed* altitude of a limb;
3. The *apparent* altitude of the limb;
4. When corrected for refraction and parallax, the *true altitude* of the limb;
5. The *true* altitude of the center.

Except with the sea-horizon, the observed and apparent

altitudes are the same. For the fixed stars, and for the planets when their semidiameters are not taken into account, the altitudes of the limb and the center are the same. For the moon, see Art. 102.

Unless otherwise stated, the *true altitude of the center* is the altitude which enters into the following problems, and is denoted by *h*.

154. The *hour-angle* of a body can be found when the local time and longitude, or the Greenwich time, are given. (Arts. 145, 146.) For noting the time of an observation, a clock, chronometer, or watch is used; at sea, only the last two; but it will be necessary to know how much it is too fast or too slow of the particular time required.

155. The *declination* of a body can be found when the Greenwich time is known.

The polar distance of a heavenly body is the arc of the declination-circle between the body and the elevated pole of the place; that is, the *north* pole, when the place is in *north* latitude; the *south* pole, when it is in *south* latitude. If

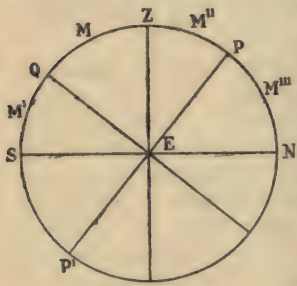


FIGURE 27

P P′ (Fig. 27) is the projection of the declination-circle through an object, M;
P, the north pole;
P′, the south pole;
E Q, the equator; then the polar distances,

$$P\,M = P\,Q - Q\,M = 90^\circ - d.$$
$$P'\,M = P'\,Q + Q\,M = 90^\circ + d.$$

That is, the polar distance is $90^\circ - d$ or $90^\circ + d$, according as the pole from which it is reckoned is N. or S. This, however, is regarding declination, like the latitude, as positive when N., negative when S.

To avoid, however, the double sign in the investigation of the formulas of Nautical Astronomy, we shall in most cases consider the declination, which is of the *same* name as the latitude, as *positive*, and that which is of a *different* name from the latitude, as *negative;* hence the polar distance will be represented by

$$p = 90^\circ - d.$$

When the declination is of a different name from the latitude, we have *numerically*

$$p = 90^\circ + d.$$

ALTITUDE AND AZIMUTH

156. ***To find the altitude and azimuth of a heavenly body at a given place and time.*** (Time-Azimuth.)

Solution. Find the declination of the body and its hour-angle, at the given time. (Arts. 145, 146.)

Then in the spherical triangle P M Z Fig. 28), we have

given $\quad P Z = 90^\circ - L,$
$\quad P M = 90^\circ - d,$
$\quad Z P M = t,$

to find
$\quad Z M = 90^\circ - h,$
$\quad P Z M = Z.$

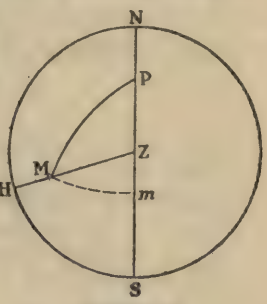


FIGURE 28

By SPH. TRIG., if in the triangle A B C (Fig. 29), we have given b, c, and A to find a and B, we have

$$\left.\begin{aligned} \tan \phi &= \tan b \cos A, \\ \cos a &= \frac{\cos (c - \phi) \cos b}{\cos \phi}, \\ \cot B &= \frac{\sin (c - \phi) \cot A}{\sin \phi}, \end{aligned}\right\}$$

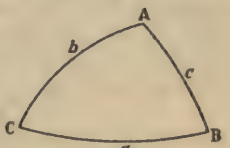


FIGURE 29

which, by substituting the corresponding parts of the triangle P Z M, give

$$\left.\begin{aligned} \tan \phi &= \cot d \cos t, \\ \sin h &= \frac{\sin (\phi + L) \sin d}{\cos \phi}, \\ \cot Z &= \frac{\cos (\phi + L) \cot t}{\sin \phi} \end{aligned}\right\} \qquad (68)$$

If we put $\phi = 90^\circ - \phi'$, these become

$$\left.\begin{aligned} \tan \phi' &= \tan d \sec t, \\ \sin h &= \frac{\cos (\phi' - L) \sin d}{\sin \phi'}, \\ \cot Z &= \frac{\sin (\phi' - L) \cot t}{\cos \phi'}, \end{aligned}\right\} \qquad (69)$$

which afford the convenient precept, *ϕ' has the same name, or sign, as the declination, and is numerically in the same quadrant as t.*

157. When $t = 6^h$, $\phi' = 90°$, and the 3d of (69) assumes an indeterminate form. But from the 1st we have

$$\cot\ t = \frac{\tan d}{\tan \phi' \sin t};$$

which, substituted, gives

$$\cot\ Z = \frac{\sin\ (\phi' - L) \tan d}{\sin \phi' \sin t}, \qquad (70)$$

which may be used when t is near 6^h.

158. h is the *true* altitude of M. If the *apparent* altitude is required, the parallax (Art. 97) must be subtracted, and the refraction (Art. 84) added.

Z is the *true* bearing, or azimuth, of the body, reckoned from the N. point of the horizon in *north* latitude, and from the S. point in *south* latitude. It is generally most convenient to reckon it as positive toward the *east*, which will require in the above formulas $-Z$ for Z, since t is positive when west. Restricting, however, Z numerically to 180°, it may be marked E. or W., like the hour-angle.

159. In Fig. 28, if $M\,m$ be drawn perpendicular to the meridian, then

$$P\,m = \phi = 90° - \phi',$$
$$Z\,m = (\phi + L) - 90° = L - \phi';$$

or, ϕ is the polar distance of m,
ϕ', its declination,

$L - \phi$, its zenith distance, positive, or of the same name as the latitude, toward the equator. A convenient precept is to mark it N. or S., according as the zenith is N. or S. of the point m. m falls on the same side of the zenith as the equator when $Z > 90°$; at the zenith when $Z = 90°$; and on the same side as the elevated pole when $Z < 90°$. It falls between P and Z only when t and Z are both less than 90°.

160. In the case of *Polaris*, (α Ursæ Minoris) whose polar distance is small, the more convenient formulas derived from (68) will be, since p and ϕ are small,

$$\phi = p \cos t,$$

(which gives ϕ within $0''.5$)

$$\sin h = \sin\ (L + \phi) \frac{\cos p}{\cos \phi},$$

$$\tan Z = \frac{\tan p \sin t \cos \phi}{\cos\ (L + \phi)};$$

or approximately,

$$h = L + \phi,$$
$$Z = p \sin t \sec (L + \phi).$$

Z is a maximum, or the star is at its greatest elongation, when the angle Z M P (Fig. 28), or Z n P (Fig. 37), is 90°. We then have

$$\sin Z = \sin p \sec L,$$

or nearly

$$Z = p \sec L.$$

161. ***To find the altitude of a heavenly body at a given place and time, when its azimuth is not required.***

Solution. The 1st and 2d of (68) or (69) may be used; or, by SPH. TRIG.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

we have $$\sin h = \sin L \sin d + \cos L \cos d \cos t. \qquad (71)$$

which, since $\cos t = 1 - 2 \sin^2 \frac{1}{2} t$,

reduces to $$\sin h = \cos (L - d) - 2 \cos L \cos d \sin^2 \tfrac{1}{2} t. \qquad (72)$$

$(L - d)$ becomes *numerically* $(L + d)$ when L and d are of different names.

Table 44 contains for the argument t in column P.M. the log $\sin \frac{1}{2} t$ in the column of *sines;* which, doubled, is log $\sin^2 \frac{1}{2} t$. It is well to note this; for mistakes are often made by regarding the logarithms in this table as log sin, log cos, etc., of t instead of $\frac{1}{2} t$.

Second Solution. By the haversine formula

From (71) we have

$$\cos z = \sin L \sin d + \cos L \cos d \cos t$$

but $$\cos t = 1 - \text{versin } t$$

$$\therefore 1 - \text{versine } z = \sin L \sin d + \cos L \cos d \ (1 - \text{vers } t)$$

or $$1 - \text{vers } z = \sin L \sin d + \cos L \cos d - \cos L \cos d \text{ vers } t$$

$$-\text{vers } z = -1 + \cos (L \mp d) - \cos L \cos d \text{ vers } t$$

divide by 2

$$\text{hav } z = \text{hav } (L \mp d) + \cos L \cos d \text{ hav } t$$

Let $$\cos L \cos d \text{ hav } t = \text{hav } \theta$$

then $$\text{hav } z = \text{hav } (L \mp d) + \text{hav } \theta \qquad (73)$$

(See Table 45, Bowditch.)

162. ***To find the azimuth of a heavenly body from its observed altitude at a given place.*** (Altitude-Azimuth.)

Solution. In this the Greenwich time of the observation must be known sufficiently near for finding the declination of the body. The observed altitude must be reduced to the true altitude. Then in the triangle P Z M we have given the three sides to find the angle P Z M.

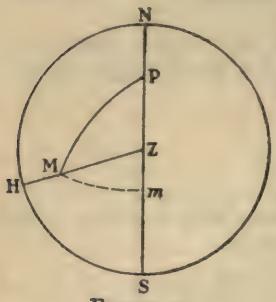


FIGURE 30

In the triangle A B C, putting $s = \frac{1}{2}(a+b+c)$, we have, by Spherical trigonometry

$$\cos \tfrac{1}{2} B = \sqrt{\left(\frac{\sin s \sin (s-b)}{\sin a \sin c}\right)}$$

For the triangle P Z M,

$B = Z$, $a = 90° - h$, h being the true altitude,
$b = p$, the polar distance,
$c = 90° - L$, the co-latitude,
$s = 90° - \frac{1}{2}(L + h - p)$,
$s - b = 90° - \frac{1}{2}(L + h + p)$,

and the formula becomes

$$\cos \tfrac{1}{2} Z = \sqrt{\left(\frac{\cos \tfrac{1}{2}(L+h+p) \cos \tfrac{1}{2}(L+h-p)}{\cos L \cos h}\right)};$$

or, if we put $s' = \frac{1}{2}(L+h+p)$,

$$\cos \tfrac{1}{2} Z = \sqrt{\left(\frac{\cos s' \cos (s'-p)}{\cos L \cos h}\right)}. \qquad (74)$$

which accords with Bowditch's rule, Art. 355.

In a similar way we may find from the formula

$$\sin \tfrac{1}{2} B = \sqrt{\left(\frac{\sin (s-a) \sin (s-c)}{\sin a \sin c}\right)},$$

$$\sin \tfrac{1}{2} Z = \sqrt{\left(\frac{\cos \tfrac{1}{2}(\text{co } L + h + d) \sin \tfrac{1}{2}(\text{co } L + h - d)}{\cos L \cos h}\right)},$$

in which $\text{co } L = 90° - L$;

or, if we put $s'' = \frac{1}{2}(\text{co } L + h + d)$,

$$\sin \tfrac{1}{2} Z = \sqrt{\left(\frac{\cos s'' \sin (s''-d)}{\cos L \cos h}\right)}. \qquad (75)$$

since $\text{hav } Z = \sin^2 \frac{1}{2} Z$

then $\text{hav } Z = \cos s' \cos s' - p, \sec L, \sec h \qquad (75a)$

(74) is preferred when $Z > 90°$; (75) when $Z < 90°$.

If the body is in the visible horizon, then nearly

$$h = -(36' 30'' + \text{the dip}).$$

Second Solution. In the astronomical triangle P Z M

$$\frac{\sin Z}{\sin t} = \frac{\sin (90° - d)}{\sin (90° - h)}$$

therefore $$\sin Z = \sin t \cos d \sec h. \qquad (76)$$

This is a very useful formula for obtaining the azimuth when used with the haversin formula in problems of the new navigation.

163. If the bearing of the body is observed with a compass at the same time that its altitude is measured, or if the bearing is observed and the local time noted, the *error* of the compass can be found. For the true azimuth, or bearing, of the body can be found from its altitude (Art. 162), or from the local time (Art. 156); and the compass error is simply the difference of the *true* and *compass* bearings of the same object, determined simultaneously if the object is in motion. It is marked *E.* when the true bearing is to the *right* of the compass bearing, *W.* when the true bearing is to the *left* of the compass bearing. (Bowditch, Art. 344.)

[In the triangle P M Z (Fig. 30), representing the positive angle, P M Z, by M, we have by Napier's "Analogies"

$$\left.\begin{aligned} \tan \tfrac{1}{2}(Z - m) &= \cot \tfrac{1}{2} \text{ t} \sin \tfrac{1}{2}(L - d) \sec \tfrac{1}{2}(L + d) \\ \tan \tfrac{1}{2}(Z + m) &= \cot \tfrac{1}{2} \text{ t} \cos \tfrac{1}{2}(L - d) \operatorname{cosec} \tfrac{1}{2}(L + d) \end{aligned}\right\} \qquad (77)$$

The Azimuth Tables (Hydrographic Office, Pub. No. 71), were computed by means of (77). From them the azimuth of any heavenly body whose declination does not exceed 23° nor the latitude 61° may be found, its hour-angle, declination, and the latitude being known. H. O. Pub. 120, tabulated for latitudes from 0° to 70° and declinations from 24° to 70° is also very useful. In both tables the azimuths are given at intervals of 10 minutes of hour-angle.

These tables afford a very simple as well as accurate method of ascertaining the azimuth, and are therefore specially valuable for the usual compass-work on board vessels.]

EXAMPLE 1: In Lat. 49° 30′ N., find the azimuth of a celestial body whose declination is 19° 08′ 06″ N., and whose H. A. is $+ 2^h 22^m 51^s$. (Use Azimuth Tables.)

Lat.....	49° N.	Lat.....	50° N.	49° 30′
Dec.....	19° 08′.1 N.	Dec.....	19° 08.1 N.	19° 08.1
H. A. $2^h 20^m$....	124° 15′		125° 11′	124° 43′
2 30.....	121° 22′		122° 17′	121 50
Change per minute.				− 17′.3

$$2^m\ 51^s = 2^m.85 \times 17'.3 = 49'$$
$$124^\circ\ 43' - 49' = 123^\circ\ 54'$$

164. The *amplitude* of a heavenly body when in the true horizon is its distance from the east or the west point, and is marked N. or S., according as it is north or south of that point; it is, then, the complement of the azimuth.

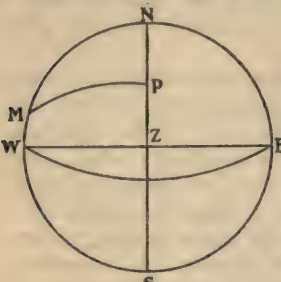


FIGURE 31

To find the amplitude of a heavenly body when in the horizon of a given place.

Solution. Let the body be in the horizon at M (Fig. 31), A = W M, its amplitude. The triangle P M N is right-angled at N, and there are given

$$P\,N = L,$$
$$P\,M = 90^\circ - d,$$

to find

$$N\,M = Z = 90^\circ - A.$$

We have $\cos P\,M = \cos P\,N \cos N\,M,$

or $\sin d = \cos L \cos Z,$

whence $\cos Z = \sin A = \sin d \sec L,$ (78)

as in Bowditch, Art. 348. By (78) A is N. or S. like the declination.

As the equator intersects the horizon of any place in the east or west points, it is plain that the star will rise and set *north* or *south* of these points, according as its declination is N. or S.

Table 39 (Bowditch) contains the amplitude, A, for each 1° of latitude up to 70°, and each ½° of declination to 30° computed by (77). The convenience of this table, in the case of the sun, is the only reason for introducing amplitudes. It is generally best to express the bearing of an object by its azimuth.

In this problem the body is supposed to be in the true horizon, or about (36′ + the dip) above the visible horizon. Hence the rule to "observe the bearing of the sun, when its center is about one of its diameters above the visible horizon." Or, the body may be observed in the visible horizon and a correction (Table 40, Bowditch) applied for the vertical displacement. Art. 348 (Bowditch).

EXAMPLE 1: January 25, 1918, in Lat. 49° 30′ 00″ S., Lon. 102° 39′ 15″ E., a watch read $2^h\ 33^m\ 13^s$ P.M., C − W $5^h\ 09^m\ 23^s$, chronometer slow $1^m\ 57^s.2$. Find the sun's altitude and azimuth.

$\lambda = 102°\ 39'\ 15''\ E = -\ 6^h\ 50^m\ 37^s$ (Table 45, Bowditch) by inspection.

Time

	h	m	s
W.	2	33	13
C − W.	5	09	23
C	7	42	36
c. c.	+	1	57.2
G. M. T.	19	44	33.2
Jan. 24	= $19^h.74$		

	h	m	s
G.M.T.	19	44	33.2
Eq. t.	−	12	19.2
G. A. T.	19	32	14.0
λ E.	= 6	50	37.0
L. A. T.	= 2	22	51.0

t = 35° 42′ 45″ (Tab. 45)

Eq. of T.

	m	s	H. D.
At, 18^h,	−12	18.2	$-0^s.6$
Corr.		−1.0	$1^h.74$
Eq. t. $19^h.74$	−12	19.2	$-1^s.04$

	°	′	″
t =	35	42	45
d =	19	08	06
ϕ' =	23	08	17
L =	49	30	00 S.
$\phi' - L$ =	−26	21	43 N.
h =	48	22	16.
Z = S. 123	53	20 W.	
or	(303°	53′	20″)

Declination

	°	′		
At, 18^h	−19	09.1		H. D. +0′.6
Corr.	+	1′.0	G. M. T.	$=1^h.74$
Dec. $19^h.74$	= S. 19°	08′.1	Corr.	+1′.04

sec	0.09047			cot	0.14333
tan	9.54029	sin	9.51561		
tan	9.63076	cosec	0.40567	sec	0.03642
		cos	9.95231	sin	−9.64742
		sin	9.87359		
				cot	−9.82717

NOTE. — As a check, a hasty inspection of H. O. azimuth tables No. 71, with Lat. 49° S., Dec. 19° S. H. A. $2^h\ 20^m$ P.M. gives Z = S 124° 22′ W. or see example under Art. 163.

EXAMPLE 2: July 20, 1918, in Lat. 38° 19′ 20″ N., Lon. 150° 15′ 30″ E., a watch read $5^h\ 58^m\ 20^s$ A.M., C − W $2^h\ 01^m\ 23^s$, chronometer fast $1^m\ 25^s$. The sun's bearing by compass at this time was N. 75° E. Variation from chart 4° W. (vessel on S.W. course). Find the deviation of the compass on the vessel's heading at the time of observation.

$\lambda = 150°\ 15'\ 30'' = 10^h\ 01^m\ 02^s$ E.

Time	h	m	s
W.	5	58	20
C − W.	2	01	23
C	7	59	43
c. c.	−	1	25
G. M. T. July 19	7	58	18
	= $7^h.97$		

Eq. of time	m	s	H.D.
At 6^h	− 6	04.6	$-0^s.2$
Corr.	−	0.4	G.M.T. $1^h.97$
Eq. t. $7^h.97$ =	− 6	05	Corr. − .39

Declination	°	′	H.D.
At 6^h	+20	54.8	− 0′.5
Corr.	−	0.9	G. M. T. $1^h.97$
Dec. $7^h.97$	N. 20	53.9	Corr. − 0′.9

	h	m	s
G. M. T.	7	58	18
Eq. t.	−	6	05
G. A. T.	7	52	13
λ E.	10	01	02
L. A. T.	= 17	53	15
	t = −6	06	45
	or E. 91°	41′	15″

	°	′	″					
t =	−91	41	15		sec	−1.53094	cosec	0.00019
d =	20	53	54	N.	tan	9.58188	tan	9.58188
ϕ' =	+94	24	37		tan	−1.11282	cosec	0.00129
L =	38	19	20	N.				
$\phi' - L$ =	+56	05	17				sin	9.91902
Z =	N. 72° 21′ 39″ E.						cot	9.50238
	or (72° 21′ 39.)							

True bearing	= N. 72° 21′ 39″ E.
Variation	4 00 00 W.
Magnetic bearing	N. 76 21 39 E.
Compass bearing	N. 75 00 00 E.
Deviation on S. W.	= 1° 22′ E.

EXAMPLE 3: February 1. 1918, at sea, in Lat. 49° 30′ N., Lon. 135° 16′ W., a deck watch read $8^h\ 45^m\ 30^s$ P.M., C − W $9^h\ 04^m\ 07^s$, chronometer fast $1^h\ 37^s$. Find the true altitude and azimuth of the star Sirius (α Can. Maj.) (haversine formula). (Table 45, Bowditch.)

Time	h	m	s
W.	8	45	30
C − W.	9	04	07
C	5	49	37
c. c.		1	37
G. M. T., Feb. 1	17	48	00
R. A. M. S.	20	46	27.1
G. S. T.	14	34	27.1
λ W.	9	01	04
L. S. T.	5	33	23.1
*'s R. A.	6	41	34.4
t =	1	08	11.3 E.
or	17°	02′	49″

Star's R. A. and Dec.	h	m	s
*'s R. A.	6	41	34.4
*'s Dec.	S. 16°	36′	.4

R. A. M. S.	h	m	s
At G. M. N., Feb. 1	20	43	31.7
Corr. G. M. T.		+2	55.4
R. A. M. S.	20	46	27.1

$t = 1^h\ 08^m\ 11^s.3$
$L = 49°\ 30′\ 00″$ N.
$d = 16°\ 36′\ 24″$ S.
$\theta =$
$L \curvearrowleft d = 66°\ 06′\ 24″$
$z = 67\ 48\ 30$
$h = 22\ 11\ 30$
$Z =$ N. 162° 20′ 18″ E.

	Altitude		*Azimuth*
hav	8.34179	sin	9.46709
cos	9.81254		
cos	9.98150	cos	9.98150
hav	8.13583		
nat. hav	.01367		
nat. hav	29748		
nat. hav	31115	cosec	0.03342
	(or S. 17° 39′ 42″ E.)	sin	9.48201

EXAMPLE 4: May 20, 1918, in Lat. 40° 15′ S., Lon. 107° 15′ W., observed with a sextant, the altitude of the sun's lower limb 10° 15′ 20″ at the same instant it center bore by compass N. 34° E. Watch time of observation $8^h\ 16^m\ 59^s$ A.M., C − W $7^h\ 02^m\ 24^s$., chronometer slow $3^m\ 53^s$, I. C. + 3′ 20″, height of eye 18 feet. Find the sun's azimuth and the deviation of the compass. Variation from chart 20° E. (vessel heading N.E.).

Time				Altitude				Corr. to Alt.			Declination			
	h	m	s		°	′	″		′	″		°	′	
W	8	16	59	☉	10	15	20	I. C.	+ 3	20	At 2^h	+19	53.1	H. D. +0′.5
C–W	7	02	24	Corr.	+	9	58	S. D.	+15	50	Corr.	+	0.7	G. M. T. = $1^h.39$
C	3	19	23	h	= 10	25	18	Dip	− 4	09		N. 19	53.8	Corr. +0′.69
c. c.	+	3	53					p. & r.	− 5	03	p	=109	53.8	
G.M.T. May 20	3	23	16					Corr.	+ 9	58				
	= $3^h.39$							or (Tab. 46, Bowd.)	= 6′ 38″					
								I. C. =	+3	20				
								Corr. =	+9	58				

	°	′	″			
h =	10	25	18	sec	0.00723	
L =	40	15	00	sec	0.11734	
p =	109	53	48			
2 s =	160	33	56			
s =	80	16	58	cos	9.22732	
s − p =	−29	36	50	cos	9.93921	
				hav z	9.29110 = N. 52° 28′ 47. E.	

True bearing of sun	N. 52° 28′ 47″ E.	
Variation	20 00 00 E.	
Magnetic bearing	N. 32 28 47 E.	
Compass bearing	N. 34 00 00 E.	
Deviation on N. E. heading	=	1° 31′ W.

EXAMPLE 5: September 20, 1918, in Lat. 30° 25′ N., Lon. 50° 16′ W., about $6^h\ 02^m$ P.M. local apparent time, the compass bearing of the sun's center, taken when the center was estimated to be one diameter above the visible horizon, was S. 80° W. (260°). Find the compass error.

	h	*m*	*s*
L. A. T.	6	02	00
λ W.	+3	21	04
G. A. T.	9	23	04
Eq.t.	−	6	29
G. M. T.	9	16	35
Sept. 20	$9^h.28$		

Eq. t. 9^h $+6^m\ 29^s$

	°	′
Dec. 8^h	+1	10.8
		−1.3
Dec. $9^h.28$ = N.	1	09.5

H. D. −1′.0

G. M. T. $1^h.28$

Corr. −1′.28

	°	′	″		
d =	1	09	30	sin	8.30567
L =	30	25	00	sec	.06431
A = W.	1	20	35 N.	sin	8.36998

True amplitude = W. 1° 20′ 35″ N.
Compass amplitude = W. 10 00 00 S.
Compass error 11° 20′ 35″ E.

By inspection Table 39 (BOWD.) amplitude = W. 1° 18′ N.

AZIMUTH OF A TERRESTRIAL OBJECT

165. In conducting a trigonometric survey, it is necessary to find the azimuth, or true bearing, of one or more of its lines, or of one station from another. Thence, by means of the measured horizontal angles, the azimuths of other lines or stations can be found; and, still further, a meridian line can be marked out upon the ground, or drawn upon the chart.

For example, suppose at a station, A, the angles reckoned to the *right* are
B to C, 48° 15′ 35″; C to D, 73° 37′ 16″; D to E, 59° 45′ 20″; and that the azimuth of D is N. 35° 16′ 15″ E.; the azimuths of the several lines are

$A\,B$,	N.	86°	36′	36″	W.	(273°	23′	24″)
$A\,C$,	N.	38	21	1	W.	(321	38	59)
$A\,D$,	N.	35	16	15	E.	(35	16	15)
$A\,E$,	N.	95	1	35	E.	(95	01	35)

If upon the chart a line be drawn, making with $A\,B$ an angle of 86° 36′ 36″ to the *right*, or with $A\,D$ an angle of 35° 16′ 15″ to the *left*, it will be a meridian line.

Or, if a theodolite or compass be placed at A in the field, and its line of sight, through the telescope or sight-vanes, be directed to D, and the readings noted, and then the line of sight be revolved to the *left* until the readings differ 35° 16′ 15″ from those noted, it will be directed *north*. If a stake or mark be be placed in that direction, it will be a *meridian mark* north from A.

166. If the azimuth of a terrestrial object is known, it may be conveniently used in finding the magnetic declination, or variation of the compass. For, let the bearing of the object be observed with the compass ashore — the difference of this magnetic bearing and the true bearing is the magnetic declination, or variation, required. It is *east* if the true bearing is to the *right* of the magnetic bearing; but *west* if the true bearing is to the *left* of the magnetic bearing.*

* This has reference to the two readings. The actual direction of the object is the same; but the true and magnetic meridians, from which the angles are estimated, are different. When the variation is *east*, the magnetic meridian is to the *right* of the true meridian; when the variation is *west*, the magnetic meridian is to the *left* of the true meridian.

It is necessary to distinguish between the magnetic bearing and the compass bearing. The latter is affected by the errors of the instrument employed and by local disturbances; the former is free from them.

167. ***To find the azimuth, or true bearing of a terrestrial object.***

Solution. Let

Z (Fig. 32) be the zenith, or place, of the observer;
O, the terrestrial object;
M, the *apparent* place of the sun, or some other celestial body;
Z = N Z O, the azimuth of O;
z = N Z M, the azimuth of M;
$\zeta = Z - z$ = M Z O, the *azimuth angle* between the two objects, or the difference of azimuth of M and O.

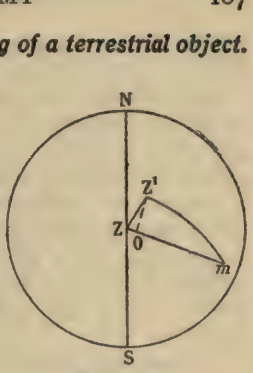


FIGURE 32

The problem requires that z and ζ be found; then we have $Z = z + \zeta$.
Or, numerically,

$Z = z + \zeta$, when the azimuth of the terrestrial object is greater than that of the celestial;
$Z = z - \zeta$, when it is less. The sign of ζ should be noted in the observations.

168. z = N Z M, the azimuth of the celestial body, may be found from an observed altitude (Art. 162), or from the local time (Art. 156). In the first case, the most favorable position is on or nearest the prime vertical; for then the azimuth changes most slowly with the altitude. In the latter, positions near the meridian may also be successfully used.

169. ζ = M Z O, the azimuth angle between the two objects, may be found in one of the following ways:

First Method. (By direct measurement.)

M Z O, being a *horizontal* angle, may be measured directly by a theodolite or a compass, by directing the line of sight of the instrument first to one of the objects and reading the horizontal circle, then to the other and reading again. The difference of the two readings is the angle required. Or, the telescope or sight-vanes of a plane table may be directed successively to the objects, and lines drawn upon the paper along the edge of the ruler in its two positions, and the angle which they form measured by a protractor.

At the instant when the observation is made of the celestial object, either its altitude should be measured, or the time noted, so as to find its azimuth simultaneously.

The instrument should be carefully adjusted and leveled.

With the compass or plane table, it is not well to observe objects whose altitudes are greater than 15°.

A theodolite can be used with greater precision than the other instruments; but the greater the altitude of the object, the more carefully must the cross-threads be adjusted to the axis of collimation, and the telescope be directed to the object.

The *error of collimation* is eliminated by making two observations with the telescope reversed either in its Vs, or by rotation on its axis. Low altitudes are generally best.

170. If the sun is used, each limb may be observed alternately; or a separate set of observations may be made for each.

To find the azimuth reduction for semi-diameter, when but one limb is observed;

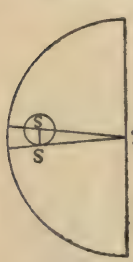


FIGURE 33

Let $h = 90° - Z\,s$ (Fig. 33) 'the altitude of the sun,
$s = S\,s$, its semi-diameter,
$s' = S\,Z\,s$, the reduction of the azimuth for the semi-diameter.

We have $$\sin S\,Z\,s = \frac{\sin S\,s}{\sin Z\,s},$$

or, since s and s' are small,

$$s' = s \sec h,$$

which is the reduction required.

The sign with which it is to be applied depends upon the limb observed.

171. If the observations are made at night, and the terrestrial object is invisible, a temporary station in a convenient position may be used, and its azimuth found. The horizontal angle between this and the terrestrial object may be measured by daylight, and added to, or subtracted from, this azimuth.

A board, with a vertical slit and a light behind it, forms a convenient mark for night observations.

The place of the theodolite should be marked, that the instrument may be replaced in the same position. But in doing this, and selecting the temporary station, it should be kept in mind that a change of the position of the instrument of $\frac{1}{3438}$ of the distance of the object may change the azimuth 1′; or of $\frac{1}{200000}$ of the distance may change the azimuth more than 1″.

172. *Second Method.* Finding the difference of azimuths of a celestial and a terrestrial object *by a sextant;* sometimes called an "*astronomical bearing.*"

Measure with a sextant the angular distance M O (Fig. 34) of the two objects, and either note the time by a watch regulated to local time, or measure simultaneously the altitude of the celestial object. Measure, also, the altitude of the terrestrial object (if it is not in the horizon), either with a theodolite which is furnished with a vertical circle, or with a sextant above the water-line at the base of the object, when there is one. Correct the readings of the instruments for index errors, and when only one limb of the sun is observed, for semidiameter.*

Observed altitudes of either object above the water-line are also to be corrected for the dip by (36) or Table 14 (Bowditch), if the horizon is free; but by (38) or Table 15 (Bowditch), if the horizon is obstructed. The altitude of the celestial object, when not observed simultaneously, may be interpolated from altitudes before and after, by means of the noted times. Or the true altitude may be computed for the local time (Art. 156 or 161) and the refraction added and the parallax subtracted to obtain the *apparent* altitude.

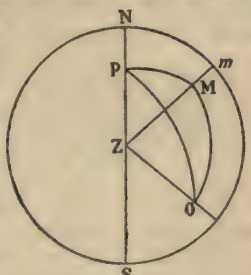


FIGURE 34

Let $h' = 90° - Z\,O$ (Fig. 32), the *apparent* altitude of O,
$H' = 90° - Z\,M$, the *apparent* † altitude of M.

$D = M\,O$, the corrected distance.

We have then in the triangle M Z O the three sides from which $\zeta = M\,Z\,O$ may be found by one of the following formulas:

1. By spherical trigonometry we have

$$\sin \tfrac{1}{2}\,\zeta = \sqrt{\frac{\sin \tfrac{1}{2}\,(D + H' - h')\,\sin \tfrac{1}{2}\,(D - H' + h')}{\cos H'\,\cos h'}}$$

or, letting $\dfrac{D + H' + h'}{2} = s$

$$\sin \tfrac{1}{2}\,\zeta = \sqrt{\frac{\sin\,(s - h')\,\sin\,(s - H')}{\cos H'\,\cos h'}} \Bigg\} \qquad (80)$$

2. Also from trigonometry,

* It is best in measuring the distance of the sun from the terrestrial object to use each limb alternately.

† The *true* altitude of M is used in finding z, its azimuth.

$$\cos \tfrac{1}{2} \zeta = \sqrt{\frac{\cos \tfrac{1}{2} (H' + h' + D) \cos \tfrac{1}{2} (H' + h' - D)}{\cos H' \cos h'}}$$

or, putting

$$\left.\begin{aligned} s &= \tfrac{1}{2} (H' + h' + D) \\ \cos \tfrac{1}{2} \zeta &= \sqrt{\frac{\cos s \cos (s - D)}{\cos H' \cos h'}} \end{aligned}\right\} \qquad (81)$$

(80) is preferable when $\zeta < 90°$; (81), when $\zeta > 90°$.

173. If O is in the true horizon, or its measured altitude above the water line equals the dip, $h = 0$, and the right triangle M m O gives

$$\cos \zeta = \cos m \, O = \cos D \sec H'; \qquad (82)$$

or, more accurately, when ζ is small (spherical trigonometry),

$$\tan \tfrac{1}{2} \zeta = \sqrt{\tan \tfrac{1}{2} (D + H') \tan \tfrac{1}{2} D - H')}. \qquad (83)$$

If the terrestrial object is in the water-line, h' is negative, and equals the dip.

174. If both objects are in the horizon, or H and h are equal and very small, we have simply

$$\zeta = D. \qquad (84)$$

In general, the result is more reliable the smaller the inclination of M O to the horizon. If M O is perpendicular to the horizon, the problem is indeterminate by this method.

175. If the terrestrial object presents a vertical line to which the sun's disk is made tangent, the reduction of the observed distance for semi-diameter is

$$s' = s \sin \mathrm{M\,O\,Z} \qquad (85)$$

and not s, the semi-diameter itself. This follows from the sun's diameter through the point of contact, O, being perpendicular to the vertical circle Z O and not in the direction of the distance O M.

As the altitude of the terrestrial object is always very small, we may find M O Z by the formula

$$\cos \mathrm{M\,O\,Z} = \frac{\sin h'}{\sin D'}$$

D' being the unreduced distance.

176. When precision is requisite, the axis of the sextant

with which the angular distance is measured must be placed at the station Z; and if the object seen *direct* is sufficiently near, the parallactic correction must be added to the sextant reading. If

Δ represent the distance of the object,
d, the distance of the axis from the line of sight or axis of the telescope, this correction is

$$p = \frac{d}{\Delta} \operatorname{cosec} 1'' = 206265'' \frac{d}{\Delta} \qquad (86)$$

It is 1′, when $\Delta = 3437.75\, d.$

177. If the distance of the terrestrial object and the difference of level above or below the level of the instrument are known, we may find its angle of elevation, nearly, by the formula

$$\tan h' = \frac{E}{\Delta},$$

Δ being the distance of the object, and
E, its elevation above the horizontal plane of the instrument.

If the object is below that plane, E and h' will have the negative sign.

NOTE. — The horizontal angle between two terrestrial objects may also be found by measuring their angular distance with a sextant, and employing the same formulas (80 to 84) as for a celestial and terrestrial object; H' and h' representing their apparent angles of elevation. Each of these may be found by direct measurement, or from the known distance and the elevation, or depression, from the horizontal plane of the observer. If the two objects are on the same level as the observer, we have simply as in (84)

$$\zeta = D.$$

EXAMPLE: July 2, 1918, at San Francisco, Cal., went ashore at Fort Point, in Lat. 37° 48′ 15″ N., Lon. 122° 28′ 30″ W., to find the true azimuth of Lime Point Lighthouse located to the northward. The deck watch read $8^h\ 30^m\ 00^s$ A.M. (120th mer. time) and the angle between the sun's center and the top of Lime Point Lighthouse measured on the sextant was 94° 31′ 00″ to the left or westward of the sun, height of eye six feet. Height of lighthouse above the shore horizon, distant 1850 yards, 0° 12′ 40″. I.C. − 1′. (Use American Ephemeris and Nautical Almanac.)

Time	h	m	s
S. F. time 120th mer. July 1	20	30	00
Lon. (120th mer.) W.	8	00	00
G. M. T. July 2	4	30	00
λ (Fort Point) W.	8	09	54
L. M. T.	20	20	06
Eq. t.	−	3	42.7
L. A. T.	20	16	23.3
t =	− 3	43	36.7
or	E. 55°	54′	10″

Declination	°	′	″	H. D.
	+23	05	21.6	−10″.51
	−		47.3	G. M. T. 4″.5
	N. 23	04	34.3	Corr. − 47″.29

Eq. of time	m	s	H. D.
	−3	40.54	− 0ˢ. 481
	−	2.16	G. M. T. 4ˢ.5
	−3	42.7	Corr. − 2ˢ.16

Alt. of sun	°	′	″
h	40	22	16
p & r +		1	02
H'	40	23	18
> Sun and lighthouse			
	°	′	
I. C.	94	31	
	−	1	
D	94	30	

Alt. of lighthouse	°	′	″
obs. h	0	12	40
I. C.	−	1	00
Dip	−	2	24
h'	0°	09′	16″

	°	′	″			
t =		55	54	10	sec 0.25135	cot 9.83057
d = N.	23	04	34	tan 9.62946	sin 9.59323	
ϕ' N.	37	14	04	tan 9.88081	cosec 0.21819	sec 0.09900
L N.	37	48	15			
$\phi' - L$ −	0	34	11	cos 9.99998		sin − 7.99751
h	40	22	16	sin 9.81140		
Z = N.	90	29	04 E.			cot − 7.92708

	°	′	″
D	94	30	00
h'	0	09	16
H'	40	23	18
$2s$ =	135	02	34
s =	67	31	17
$s - H'$	27	07	59
$s - h'$	67	22	01

sec 0.00000
sec 0.11823
sin 9.65902
sin 9.96520

	°	′	″
True azimuth of sun	N. 90	29	04 E.
Horizontal sextant angle sun to lighthouse	− 96	02	40
Azimuth of lighthouse	− 5	33	36
or	354°	26′	24″

(Tab. 45) hav. ζ = 9.74245 = 96° 02′ 40″

HOUR-ANGLE AND LOCAL TIME

178. ***To find the hour-angle of a heavenly body in the horizon.***

Solution. In the diagram of the last problem,

$$\text{M P Z} = t, \text{ the hour-angle;}$$

and in the triangle P M N are given

$$\left.\begin{aligned} \text{P N} &= L, \\ \text{P M} &= 90^\circ - d, \end{aligned}\right\} \text{ to find } \quad \text{P M N} = 180^\circ - t.$$

We have $$\cos \text{M P N} = \frac{\tan \text{P N}}{\tan \text{P M}},$$

whence $$\cos t = -\tan d \tan L. \tag{87}$$

179. From this it is apparent that when the latitude and declination have the same name, $t > 6^h$, and consequently that $2t$, or the time that the body is above the true horizon, $> 12^h$; and when the latitude and declination are of different names, $t < 6^h$ and $2t < 12^h$.

$2t$ is an interval of *sidereal* time for a fixed star, of *apparent* time for the sun.

In the case of the sun, t would be the apparent time of sunset, were the refraction and dip nothing, and $(24^h - t)$ would be the apparent time of sunrise.

Table 10 (Bowditch) contains t for different values of L and d.

180. ***To find the hour-angle of a heavenly body at a given place, and thence the local time, when the altitude of the body and the Greenwich time are known.***

Solution. Find the declination of the body for the Greenwich time, and reduce the observed altitude to the true altitude. Then in the triangle P Z M (Fig. 35) are given

$$\begin{aligned} \text{P Z} &= 90^\circ - L, \\ \text{P M} &= p, \\ \text{Z M} &= 90^\circ - h, \end{aligned}$$

to find $\text{Z P M} = t$.

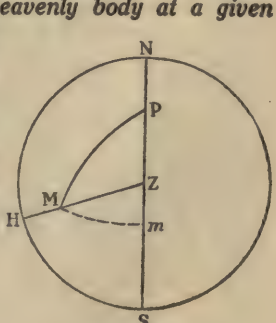


FIGURE 35

For the triangle A B C (Fig. 36), we have (by spherical trigonometry)

$$\sin \tfrac{1}{2} \text{A} = \sqrt{\left(\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}\right)}$$

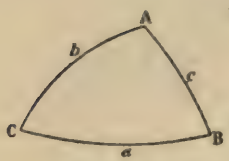


FIGURE 36

in which, putting $A = t$

$$a = 90^\circ - h,$$
$$b = p,$$
$$c = 90^\circ - L.$$

we have
$$s - b = 90^\circ - \tfrac{1}{2}(L + p + h),$$
$$s - c = \tfrac{1}{2}(L + p - h),$$

and

$$\sin \tfrac{1}{2} t = \sqrt{\left(\frac{\cos \tfrac{1}{2}(L + p + h) \sin \tfrac{1}{2}(L + p - h)}{\cos L \sin p}\right)};$$

or, if we put

$$s' = \tfrac{1}{2}(L + p + h),$$

$$\sin \tfrac{1}{2} t = \sqrt{\left(\frac{\cos s' \sin (s' - h)}{\cos L \sin p}\right)}. \qquad (88)$$

A trigonometric function which simplifies the solution of many problems in nautical astronomy and also in plane trigonometry is defined by the following relation:

$$\text{hav. A} = \tfrac{1}{2} \text{ vers. } A = \tfrac{1}{2}(1 - \cos \text{A}) = \sin^2 \tfrac{1}{2} \text{A}.$$

To afford the maximum facility in carrying out the processes of solution, the values of the natural haversine and its logarithm are given together in Table 45, Bowditch, for all values of angle from 0° to 360°, expressed both in arc and in time. When the body is east of the meridian read the value of the hour-angle from the bottom of the page. When the body is west of the meridian read the H. A. from the top of the page.

From Table 44 (Bowditch), we may take t directly from column P.M., corresponding to the log sin $\frac{1}{2} t$.

t is $-$ when the body is east of the meridian.

When the object is the sun west of the meridian, t is the apparent solar time; when the sun is east of the meridian, $(24^h - t)$ is numerically the apparent time.

When the object is the moon, a planet, or a star, we have (Art. 148), denoting its R. A. by a,

the *sidereal time* $= a + t$,

and the *mean time* $= a - S_0' + t$,

in which S_0' is the "right ascension of the mean sun." Or the sidereal time may be converted into mean time by one of the other methods of Art. 143.

[The Sunrise and Sunset Tables published in the General Tide Tables (issued by Coast Survey) were computed by applying the equation of time to the local apparent times found by

(88), assuming $h = -56' 08''$ (ref. $-36' 29''$; S. D. $-16'$; parallax, $+9''$; dip $-3' 48''$) for latitudes between 76° N. and S.

From them the local mean time of sunrise and sunset may be found, the declination and latitude being known. These tables also contain the time of moonrise and moonset at the principal American ports. Tables giving the time of sunrise, moonrise, sunset and moonset for places all over the world are issued by the Almanac Office. For computing the time of sunrise and sunset in these tables the Almanac Office use 90° 50′ for the sun's zenith distance. For the moon they use 90° 50′ minus the moon's parallax. The formula then gives the moon's H. A. to which is applied the moon's right ascension with a positive sign, and the R. A. M. S. with a negative sign, in order to obtain the time of moonrise and moonset.]

181. By the formula

$$\cos \tfrac{1}{2} A = \sqrt{\left(\frac{\sin s \sin (s-a)}{\sin b \sin c}\right)}, \quad \text{Sph. Trig.}$$

we may obtain for the triangle P Z M (z being the zenith distance),

$$\cos \tfrac{1}{2} t = \sqrt{\left(\frac{\sin \tfrac{1}{2} (\text{co } L + p + z) \sin \tfrac{1}{2} (\text{co } L + p - z)}{\cos L \sin p}\right)},$$

or, putting

$$\left.\begin{aligned} s &= \tfrac{1}{2} (\text{co } L + p + z, \\ \cos \tfrac{1}{2} t &= \sqrt{\left(\frac{\sin s \sin (s-z)}{\cos L \sin p}\right)}, \end{aligned}\right\} \qquad (89)$$

(89) is preferable to (88) when t considerably exceeds 6^h, which may be the case in high latitudes.

If $L = 90°$, the horizon and equator coincide, and $p + h = 90°$ and $p = z$; so that both (88) and (89) become indeterminate. In very high latitudes, then, these equations approach the indeterminate form, and it is impracticable to find with precision the local time from an observed altitude.

So also if $d = 90°$, the star is at the pole and $L = h$; and the problem is indeterminate. A great declination is therefore unfavorable.

182. If the object is in the visible horizon (rising or setting), $h = -(36' + \text{dip})$ nearly. With the sun, the instants when its upper and lower limbs are in the horizon may be noted, and the mean of the two times taken as the time of rising or setting of its center. The irregularities of refraction would affect nearly alike the dip and the apparent position of the sun.

183. If the time at which the altitude is observed is noted by a watch, or chronometer, we may readily find how much the watch or chronometer is fast or slow of the local time. (Art. 229.) For, let

C be the time noted,

T, the local time deduced from the observation:

$c = T - C$ will be the *correction* of the watch or chronometer to reduce it to *apparent* time, when T is the local *apparent* time; to *mean* time, when T is the local *mean* time; or to *sidereal* time, when T is the local *sidereal* time.

184. The observed altitude is affected by errors of observation, errors of the instrument, and errors arising from the circumstances in which the observation is made; such as irregularities of refraction affecting both the position of the body and the dip of the horizon. Errors of the first class are diminished by taking a number of observations. Thus several altitudes may be observed, and the time of each noted; and the mean of the altitudes taken as corresponding to the mean of the times, so far as the rate at which the body is rising or falling can be regarded as uniform during the period of observation. This period should then be brief.

185. We may easily find how much a supposed error of 1′ in the altitude will affect the resulting hour-angle, by dividing the difference of two of the noted times by the difference in *minutes* of the two corresponding altitudes.

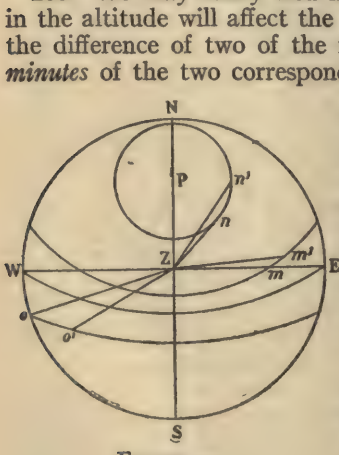


FIGURE 37

The effect will evidently be least when the body is rising or falling most rapidly. This will be the case when its diurnal circle makes the smallest angle with the vertical circle. An inspection of the diagram (Fig. 37) shows that this is the case when the object is nearest the prime vertical, or bears most nearly *east* or *west*.

Thus Z n being tangent to the diurnal circle $n\,n'$, the angle which it makes with it is 0; and is therefore less than the angle which any other vertical circle, as Z n', makes with $n\,n'$.

The diurnal circle $m\,m'$ makes a smaller angle with Z m, the prime vertical, than with any other vertical circle, as Z m'.

The diurnal circle $o\,o'$ makes a smaller angle with $Z\,o$ than with $Z\,o'$.

The diurnal circles make right angles with the meridian; so that at the instant of transit, the change of altitude is 0.

186. At sea, and to a less extent on the land, the latitude is uncertain. To ascertain the effect of an error of 1′ in the assumed latitude, the hour-angles may be found for two latitudes separately, differing, say, 10′; and the difference of these hour-angles divided by 10.

This is an essential feature of Sumner's method, which will be explained hereafter. This method will also show that an error in latitude least affects the deduced hour-angle when the body is nearest the prime vertical.

EXAMPLES:

EXAMPLE 1: March 20, 1918, in Lat. by D. R. 41° 15′ S., Lon. 86° 35′ W., the sextant altitude of the sun's lower limb was 18° 20′ 00″ I. C. – 1′ 30″, height of eye 18 feet, W. T. of observation $4^h\ 27^m\ 53^s$ P.M., C – W. $5^h\ 46^m\ 10^s$, chronometer slow $1^m\ 17^s$. At time of observation the sun's center bore by compass 277°, vessel heading S.W. Find the local mean time at vessel, the longitude and the compass error.

Time				Altitude			Corr. to Alt.			Declination			Eq. of T.			
	h	*m*	*s*	°	′	″		′	″		°	′		*H. D.*	*m*	*s*	*H. D.*
W.	4	27	53	18	20	00	S. D.	+16	05	At 10*h*	−0	12.3		+1′.0	−7	38.5	+ 0*s*.7
C − W	5	46	10	+	7	40	Dip.	− 4	09	Corr.	+	0.3	G. M. T.	0*h*.26	+	2	G. M. T. 0*h*.26
C	10	14	03	18	27	40	*p* & *r*	− 2	46		S.	0 12	Corr.	+0′.26	−7	38.3	Corr. +0*s*.18
c. c.		+1	17				I. C.	− 1	30								(+ to app. time)
G. M. T.	10	15	20				Corr.	+ 7	40	*p*	= 89° 48′						
March 20 =		10*h*.26					or (Tab. 46 Bowd.)	+9′	09″								
							I. C.	− 1	.30								
							Corr.	+7′	39″								

	°	′	″			°	′	″		
h	18	27	40			18	27	40	sec	0.02295
L	41	15	00	sec	0.12387	41	15	00	sec	0.12387
p	89	48	00	cosec	0.00000	89	48	00		
2 *s*	149	30	40			149	30	40		
s	74	45	20	cos	9.41986	74	45	20	cos	9.41986
h	18	27	40			89	48	00		
s − *h*	56	17	40	sin	9.92008	*s* − *p* =	15	02	40	cos 9.98486
				hav	9.46381					

L. A. T. = 4*h* 21*m* 08*s*.2
Eq. of t. = +7 38.3
L. M. T. = 4 28 46.5
G. M. T. = 10 15 20.0
Lon. = 5 46 33.5 W.
= 86° 38′ 23″ W.

Z = N. 73° 16′ 15″ W. = hav 9.551154
or 286° 44′ true bearing
277° 00′ compass bearing
Compass error = 9° 44′ E.

If we take L = 41° 25′ S., we shall find the l. ap. t. 4*h* 20*m* 52*s*.2; so that for a change in L = 10′ S, causes a corresponding change in t = −16*s*.

EXAMPLE 2: September 20, 1918, in Lat. 46° 16′ N., Lon. 163° 52′ E., the sextant altitude of the moon's upper limb east of the meridian was 27° 44′ 20″ I. C. + 1′ 30″, height of eye 20 feet, W. time of observation, $8^h\ 18^m\ 59^s$ P.M. C − W $1^h\ 00^m\ 41^s$, chronometer slow of G. M. T. $3^m\ 05^s$. Find the local mean time and the longitude.

Time	h	m	s
W.	8	18	59
C − W	1	00	41
C	9	19	40
c. c	+	3	05
G. M. T.	21	22	45
Sept. 19	21^h. 38		

Moon's alt.	°	′	″
	27	44	20
1st corr.	−	19	27
App.	27	24	53
2d corr.	+	52	29
	28	17	22

Corr. to alt.	′	″
I. C.	+ 1	30
Aug.	+ 0	08
S. D.	−16	42
Dip.	− 4	23
1st corr.	−19	27
2d corr. (Tab. 24, Bowd.)		
	= +52	29

	°	′	″
h	28	17	22
L	46	16	00
p	87	39	00
2 *s*	162	12	22
s	81	06	11
h	28	17	22
s − *h*	52	48	49

sec 0.16033
cosec 0.00037
cos 9.18936
sin 9.90128
2 | 9.25134
sin 9.62567

	h	m	s
t, = ☽'s H. A.	−3	19	51.7
☽'s R.A.	23	31	42.2
L. S. T.	20	11	50.5
R. A. M. S.	11	53	50.0
Sept. 20, L. M. T.	8	18	00.5
Sept. 19, G. M. T.	21	22	45.0
Lon.	10	55	15.5 E.
or	163°	48′	52″.5 E.

Moon's R. A. and Dec. S. D. and H. P.

	h	m	s	
R. A. 20^h	23	28	29	*H. D.* +140.°
Corr.	+	3	13.2	G. M. T. $1^h.38$
☽'s R. A.	23	31	42.2	corr. +193°.2

	°	′	
Dec. 20^h	+2	00.8	*H. D.* +14′.65
Corr.	+	20.2	G. M. T. 1^h.
☽'s dec.	N. 2	21	Corr. 20′.21
☽'s p	87	39	
S. D.	= 16′.7		H. P. = 61′.2

R. A. M. S.	h	m	s
G. M. N.	11	50	19.3
Corr. G. M. T.		+3	30.7
	11	53	50.0

An examination of the above problem shows that to get a good longitude it requires correctness of altitude, latitude and declination. An incorrect G. M. T. causes an equal error in longitude and also an error in getting the declination especially of the moon, whose declination

changes rapidly. As the right ascension of the moon also enters into the problem, it also is changing rapidly, so unless the G. M. T. is accurate, the results are not good. For these reasons time sights of the moon are not so accurate as those of the sun or the stars.

187. ***To find the hour-angle of a heavenly body when nearest to, or on, the prime vertical of a given place.***

Solution. If $d > L$, and with the same name, as for the body whose diurnal path is $n\,n'$ (Fig. 37), P Z n will be greatest, or nearest to 90°, when Z n is tangent to $n\,n'$, and consequently Z n p = 90°. We then have

$$\cos t = \frac{\tan p}{\cot L} = \frac{\tan L}{\tan d}. \qquad (90)$$

If $d < L$, and with the same name, as for the body whose diurnal path is $m\,m'$, the body will be on the prime vertical at m, and P Z m = 90°; whence we have

$$\cos t = \frac{\tan d}{\tan L}. \qquad (91)$$

If d and L are of different names, the diurnal circle intersects the prime vertical below the horizon, if at all, and the visible point nearest the prime vertical is in the horizon. The hour-angle of this point can be found by (87), omitting the effect of refraction,

$$\cos t = -\tan d \tan L.$$

Altitudes less than 8°, however, are to be avoided.

If $d = L$, the diurnal circle passes through the zenith, and the body would be on the meridian and prime vertical at the same instant; so that, when d and L are nearly equal, latitudes observed within a few minutes of the meridian passage of the body may be used for finding the time. It is only necessary that the change of altitude shall be sufficiently rapid.

But when the body is very near the meridian in azimuth the change of altitude is proportional, not to the intervals of time, but to the squares of the hour-angles. (Art. 199.) Hence, when the body is in such a position, the mean of several times does not correspond to the mean of the altitudes.

From the hour-angle the local time may be found by Arts. 147, 148.

When the declination of the body does not exceed 23°, t may be taken from Azimuth Tables No. 71, but for higher declinations t must be taken from H. O. Azimuth Tables No. 120; it can be taken with sufficient accuracy for ordinary purposes. (See Art. 163.)

EXAMPLES:

EXAMPLE 1: May 1, 1918, in Lat. 40° 15′ 00″ N., Lon. 65° 17′ 15″ W. Find the local mean times when the stars Arcturus (α Boötis) and Capella (α Aurigæ) are on the prime vertical or nearest to it East and West.

Arcturus (α Boötis) will be on the prime vertical
$d < L$, $\because \cos t = \cot L \tan d$

	°	′	″			
L	40	15	00 N.	cot	0.07234	
d	19	36	18 N.	tan	9.55167	
*'s t,	=	4^h	20^m 28^s.6	cos	9.62401	

	h	m	s	h	m	s
*'s t,	= −4	20	28.6	+4	20	28.6
*'s R. A.	= 14	11	58.4	14	11	58.4
L. S. T.	9	51	29.8	18	32	27.0

	h	m	s	h	m	s
R. A. M. S., May 1	2	34	25.0	2	34	25.0
Corr. λ W. (Tab. III)	+	0	42.9	+	0	42.9
Sid. time local 0^h	2	35	07.9	2	35	07.9
L. S. T.	9	51	29.8	18	32	27.0
Sid. int.	7	16	21.9	15	57	19.1
Red. (Tab. II)		1	11.5		2	36.8
L. M. T. May 1	7	15	10.4 E. of mer.	15	54	42.3 W. of mer.

Capella (α Aurigæ) will not be on the prime vertical
$d > L$ $\because \cos t = \tan L \cot d$

	°	′	″			
L	40	15	00 N.	tan	9.92766	
d	45	55	06 N.	cot	9.98607	
*'s t,	=	2^h	19^m 44^s.4	cos	9.91373	

	h	m	s	h	m	s
*'s t,	= −2	19	44.4	+2	19	44.4
*'s R. A.	5	10	39.1	5	10	39.1
L. S. T.	2	50	54.7	7	30	23.5

	h	m	s	h	m	s
Sid. time local 0^h	2	35	07.9	2	35	07.9
L. S. T.	2	50	54.7	7	30	23.5
Sid. int.	0	15	46.8	4	55	15.6
Red. (Tab. II)		0	02.6		0	48.4
L. M. T., May 1	0	15	44.2 E. of mer.	4	54	27.2 W. of mer.

An inspection of azimuth tables H. O. 71 (az. of sun) and H. O. 120 (az. of celestial bodies) taking Lat. 40° N., dec. 20°N. H. A. 4^h 20^m shows Arcturus will be on prime vertical. Taking Lat. 40 N. dec. 46°N. H. A. 2^h 20^m shows Capella will not be on the prime vertical, but bears N. 65° E. at rising and N. 65° W. at setting.

EXAMPLE 2: January 1, 1918, expecting to be in Lat. 32° 00′ N., Lon. 30° 30′ W., about sunset, and desiring to get a time sight of the planet Jupiter; to find the time when Jupiter is on the prime vertical, and also its true altitude.

(The G. M. T. is needed to correct the R. A. and Dec. From the almanac find the approx. time of the planets' transit; find from Azimuth Tables with this Lat. and Jupiter's Dec. when azimuth = 90°; this may be assumed to be a sidereal interval; correction (Table II) = mean time interval).

Times	h	m	s
Azimuth 90° =	3	38	00
Red. (Tab. II)		−	36
Approx. H. A. on P. V.	3	37	
Approx L. M. T. transit	9	20	
Approx. L. M. T. on P. V.	5	43	
Lon. W.	+2	02	
G. M. T. (approx.) Jan. 1	7	45	
	= 7^h.75		

R. A. Jupiter	h	m	s	
				H. D.
G. M. N.	4	02	29	−0^s.875
Corr.			−6.8	
G. M. T. 7^h.75	4	02	22.2	Corr. −6^s.78

Dec. Jupiter	°	′	H. D.
G. M. N.	+19	54.3	−0′.029
Corr.	−	0.2	
G. M. T. 7^h.75	N. 19	54.1	Corr. −0′.224

d =	19°	54′	06 N.
L =	32	00	00 N.
t = −	3	38	22.7
Jup. R. A. =	4	02	22.2
L. S. T.	0	23	59.5

tan 9.55874 sin 9.53200
cot .20421 cosec 0.27579
cos 9.76295
h = 39° 58′ 08″ sin 9.80779

	h	m	s
R. A. M. S, Jan. 1	18	41	18.4
Red. Lon W. (Tab. III)		+0	20.0
Sidereal time local 0^h	18	41	38.4
L. S. T.	0	23	59.5
Sid int.	5	42	21.1
Red. (Tab. II)	−	0	55.9
L. M. T. Jan. 1, when Jup. is on prime vertical	5	41	25.2

CHAPTER XI

LATITUDE

188. ***To find the latitude from an observed altitude of a heavenly body on the meridian.***

Solution. Let the diagram (Fig. 38) be a projection of the sphere on the plane of the meridian N Z S:

Z, the zenith;
N S, the horizon;
P, the elevated pole;
P P′, the axis of the sphere;
E Q, the equator;
Q Z, the declination of the zenith, and
N P, the altitude of the pole, are each equal to the latitude, *L*.

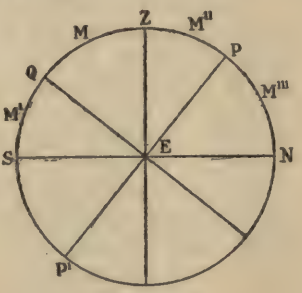


FIGURE 38

Let

M be the position of the body;
Q M = *d*, its declination;
M Z = $z = 90° - h$, its zenith distance, which it is convenient to mark N. or S., according as the zenith is north or south of the body.

From the diagram, we have Q Z = Q M + M Z

or, $$L = z + d, \qquad (92)$$

which is the general formula.

If the body is at *M′*, *numerically*

$$L = z - d;$$

if at M″, $$L = d - z;$$

or "the latitude is equal to the *sum* of the zenith distance and declination, when they are of the *same* name; to their *difference*, when of *different* names; and is of the same name as the greater."

If the body is at M‴, or below the pole,

$$Q\,M''' = 180^\circ - d, \qquad \text{and} \qquad L = 180^\circ - d - z,$$

numerically; or (92) is the correct formula, provided we use $180^\circ - d$, or the supplement of the declination, instead of the declination.

But in this case we have also from the diagram

$$L = p + h, \tag{93}$$

as in Bowditch, Art. 321.

The declination of the body must be found from the Almanac *for the time of meridian passage.* (Art. 135.) The observed altitude must be corrected for dip, refraction, etc., and the true altitude derived.

From (92) we see that an error of 1′ in the altitude will produce an error of 1′ in the resulting latitude.

Constant. — In working a meridian altitude, especially the daily noon observation of the sun, it is a convenience to arrange the terms so that all computation, excepting the application of the observed altitude, is completed beforehand; then the vessel's latitude will be known immediately after the sight has been taken, it being necessary only to add or subtract the altitude.

The noon longitude will be approximately known in advance close enough to enable the navigator to correct the declination; also the approximate meridian altitude to correct the parallax and refraction.

Then

$$\begin{aligned} \text{Lat.} &= z + d, \\ &= 90^\circ - \text{True } h + d, \\ &= 90^\circ - (\text{Obs. } h + \text{corr.}) + d, \\ &= (90^\circ + d - \text{ corr.}) - \text{obs. } h, \end{aligned}$$

in which the quantity $(90^\circ + d - \text{corr.})$ may be termed a *Constant* for the meridian altitude of the day, as it remains the same whatever the observed altitude may prove to be. The constant having been worked up before the observation is made, the latitude will be known as soon as the observed attitude is applied.

When

$d(+)$ and $> L$,	$L = h_0 - (90^\circ - d - \text{corr.})$
$d(+)$ and $< L$,	$L = (90^\circ + d - \text{corr.}) - h_0$
$d(-)$	$L = (90^\circ - d - \text{corr.}) - h_0$
At lower transit	$L = h_0 + (90^\circ - d + \text{corr.})$ or $h_0 + (p + \text{corr.})$

EXAMPLE 1: June 30, 1918, in Lat. 2° 40′ N., Lon. 105° 18′ W., by dead reckoning, the sextant altitude of the sun's lower limb on the meridian bearing N., was 69° 15′ 20″ I. C. + 3′ 20″, height of eye 20 feet. Find the latitude.

	h	*m*	*s*
G. A. T. of L. A. noon =	7	01	12
Eq. of T.	+	3	21
G. M. T. of L A. N.	7	04	33
	= 7^h.08		

Eq. of T. 7^h = − 3^m 20^s.5
(+ to app. time)

Altitude, zen. dist. Lat.					Corrections to alt.				Declination			
	°	′	″			′	″					*H. D.*
Obs. alt.	69	15	20		I. C.	+3	20		At 6^h	+23	12.1	−0′.2
Corr.	+	14	24		S. D.	+15	46		Corr.	−	0.2	G. M. T. 1^h.08
True alt.	= 69	29	44	N.	Dip.	−4	23		Dec. L. A. N. = N.	23	11.9	Corr. −0′.216
	90	00	00		p. & r.	−0	19					
Zen dist. (*z*) =	20	30	16	S.	Corr.	+14	24					
dec. (*d*) =	23	11	54	N.	or (Tab. 46 Bowd.)		= +11′	04″				
Latitude =	2	41	38	N.		I. C. =	+ 3	20				
						Corr. =	+14	24				

To find a constant which is prepared before the meridian altitude is obtained so that the sextant altitude can be applied immediately to get the noon latitude.

$$C = (90° - d - \text{corr.}); \quad L = \text{sext. alt.} - C.$$

	°	′	″	
Dec. 6^h	23	12	06	
Corr.		−	12	
L. A. N. (corr. dec.)	23	11	54	
	90	00	00	
$90° - d$	66	48	06	
Corr. (approx.)	−	14	43	
$90° - d$ − corr.	66	33	23	
Lat. D. R.	2	40	00	N.
Alt. (approx.)	= 69	13	23	

	′	″
I. C.	+ 3	20
S. D.	+15	46
Dip.	− 4	23
Corr. (approx.)	+14	43
p & r	− 0	19
Corr.	+14	24

	°	′	″	
	90	00	00	
L. A. N. (corr. dec.)	23	11	54	
$90° - d$	66	48	06	
Corr.	−	14	24	
Constant (c)	= 66	33	42	
Sext. alt.	69	15	20	
Lat.	2	41	38	N.

EXAMPLE 2: July 4, 1918, in Lat. 44° 43′ N., Lon. 150° 15′ E., by D. R. the sextant altitude of the sun's lower limb on the meridian bearing S. was 68° 07′ 30″ I. C. − 1′ 30″, height of eye, 20 feet. Find the latitude.

(Lon. = − 10^h 01^m 00^s)

	h	*m*	*s*
G. A. T. of L. A. N., July 3	13	59	00
Eq. of T.		+3	59
G. M. T. of L. A. N., July 3	14	02	59
	= $14^h.05$		

Eq. t. 14^h, -3^m 59^s (+ to app. t.)

Alt, z., Lat.	°	′	″
Obs.	68	07	30 S.
Corr.	+	9	32
True alt.	68	17	02
Zen. dist. =	21	42	58 N.
d =	22	58	12 N.
L =	44	41	10 N.

Alt. corr.	′	″
I. C.	− 1	30
Dip	− 4	23
p. & *r.*	− 0	21
S. D.	+ 15	46
Corr.	+ 9	32
	′	″
or (Tab. 46 Bowd.)	+ 11	03
I. C.	− 1	30
Corr.	+ 9	33

Declination	*H. D.*
At 14^h, +22° 58′.2	−0.2
or at L. A. N.	G. M. T. $0^h.05$
	Corr. −.010

Constant = $(90° + d - \text{corr.})$ $L = C - \text{sext. alt.}$

	°	′	″
Dec. 14^h	22	58	12
at L. A. N.	90	00	00
$90° + d$	112	58	12
Corr. (approx.)		− 9	53
$(90° + d - c)$	112	48	19
Lat. D. R.	44	43	00
Alt. (approx.) =	68	05	19

	′	″
S. D.	+15	46
I. C.	− 1	30
Dip	− 4	23
Corr. (approx.) =	+ 9	53
p & r	− 0	21
Corr.	+ 9	32

	°	′	″	
L. A. N. (corr. dec.)	22	58	12	
	90	00	00	
$90° + d$	112	58	12	
Corr.		−9	32	
Constant (c)	112	48	40	
Sext. alt.	68	07	30	
Lat.	44	41	10	N.

EXAMPLE 3: July 29, 1918, about 5^h A.M. in Lon. 85° 15′ W., the sextant altitude of the moon's lower limb on the meridian bearing N. was 50° 18′ 00″ I. C. + 2′ 00″, height of eye 20 feet. Find the latitude.

Meridian Transit

5 A.M. July 29 = 17^h, July 28.

	h	m
July 28, G. M. T. of Gr. tr.	16	32
Corr. λ W.	+	13
L. M. T. of local tr.	= 16	45
Lon. W.		5 41
G. M. T. of local tr. July 28	= 22	26
	22^h.	43

Retardation for Longitude

H. D. $2^m.2$
Lon. = $5^h.7$
Corr. +$12^m.5$
or (Tab. 11, Bowd.) = $12^m.5$

Altitudes

	°	′	″
☽	50	18	00
1st corr.	+	14	02
	50	32	02
2d corr. (Tab. 24)	+	37	03
	51	09	05 N.
z =	38	50	55 S.
d =	12	14	42 N.
Lat. =	26	36	13 S.

Correction

	′	″
I. C. +	2	00
S. D. +	16	12
Aug. +		13
Dip −	4	23
1st corr. +	14	02
Total corr. (Tab. 49 Bowd.) = +	49′	06″
I. C. +	2	00
Corr.	51	06

Declination

	°	′	H. D.
At 22^h +	12	09.5	+12 .1
Corr.		+5.2	G. M. T. $0^h.43$
Dec. N.	12	14.7	Corr. +5′.203

S. D. = 16′.2 H. P. = 59′.5

EXAMPLE 4: September 15, 1918, in Lat. 70° 00′ N., Lon. 170° 00′ E., the sextant altitude of the star Vega (α Lyræ), on the meridian bearing S. was 58° 43′, I C. − 1′ 30″, height of eye 20 feet. Find the local mean time, so as to know when to take the observation and also find the latitude.

To find the time of obs.				Altitudes				Correction		Declination and R. A.
	h	*m*	*s*		°	′	″	′	″	
R. A. M. S., Sept 15	11	34	33.1	*s sextant alt.	58	43	00			* R. A., Sept. 15 = 18^h 34^m 12^s.2
Corr. Lon. E. (Tab. III)	−	1	51.7	Corr.	−	6	29	I. C.	−1	30	
Sid. t. local o^h	11	32	41.4	*s true alt.	58	36	31 S.	Dip.	−4	23	* Dec., Sept. 15 = N. 38° 42′.7
L. S. T. = *s R. A.	18	34	12.2					r	−	36	
Sid. int. from noon	7	01	30.8	*s true z	31	23	29 N.	Corr.	−6	29	
Red. (Tab. II)		1	09.1	*s dec.	38	42	42 N.	(Tab. 46 Bowd.) =	−4	59	
L. M. T. of *s tr., Sept. 15, 7 P.M.	7	00	21.7	Lat.	70	06	11 N.	I. C.	−1	30	
								Corr.	−6	29	

(The almanac gives the meridian transit of Vega, G. M. T. of transit at Greenwich, Sept. 15, 6^h 59^m.)

EXAMPLE 5: June 1, 1918, the sextant altitude of the star Capella (α Aurigæ), on the meridian below the pole was 10° 40′ 00″ I. C. − 1′ 30″, height of eye 30 feet. Find the latitude.

Alt. P, L.				Corr. to Alt.			*dec. and p. d.			
	°	′	″		′	″		°	′	″
*'s sextant alt.	10	40	00	I. C.	− 1	30	*'s d = N.	45	55	00
Corr.	−	11	52	Dip.	− 5	22	*'s p =	44	05	00
*'s true alt.	= 10	28	08	r.	− 5	00				
*'s polar d	44	05	00	Corr.	−11	52				
Lat.	= 54	33	08N.	or (Tab. 46, Bowd.)						
					′	″				
					−10	22				
				I. C.	− 1	30				
				Corr.	−11	52				

EXAMPLE 6: January 3, 1918, in Lat. 75° S., Lon. 0°, by D. R. the sextant altitude of the sun's lower limb at lower transit was 8° 03′ 40″, I. C. − 1′ 00″, height of eye 30 feet. Find the latitude. (This transit is at local apparent midnight = $12^h + \lambda$ = January 3, 12^h.)

Alt. and Lat.				Alt. Corr.			G. A. T. of L. A. midnight =	h 12	m 00	s 00.0
							Eq. t.	+	4	36.4
							G. M. T. of L. A. midnight	12	04	36.4
								= $12^h.08$		
	°	′	″		′	″		°	′	*H. D.*
☉	8	03	40	S. D.	+16	18	Dec. 12^h	− 22	49.9	−0′.2
Corr.	+	3	34	I. C.	− 1	00	Corr.		−0.0	$0^h.08$
h =	8	07	14	Dip.	− 5	22	Dec.	S. 22	49.9	Corr. −0.02
p =	67	10	06	p. & r.	− 6	22	p = 67° 10′ 06″			
Lat. =	75	17	20 S.	Corr.	+ 3	34				

189. The common mode at sea of measuring a meridian altitude of the sun, is to commence observing the altitude 20 or 30 minutes before noon, repeating the operation until the highest altitude is attained; soon after which the sun, as seen through the sight-tube of the instrument, begins to *dip*, or descend below the line of the horizon.

It is preferable, however, to find, from A.M. observations for time and by allowing for the run of the ship in the inter-

val, the time of apparent noon by a watch, and observing the altitude at that time within 1^m or 2^m.

A meridian altitude of the moon, or a star, can be much more conveniently observed by finding beforehand the watch time of its culmination, and measuring the altitude at or very near that time.

When the sea is heavy, it is recommended to observe three or four altitudes in quick succession, within 2^m of the time of culmination.

190. If the body is changing its declination, or the observer his latitude, the *maximum* altitude is not at the instant of meridian passage; but *after*, if the body and zenith are approaching; *before*, if they are separating. Let

t be the hour-angle of this culminating point, in *minutes;*

$\Delta\, d$, the combined change * of declination and latitude in 1^m, if it is expressed in *seconds;* or in 1^h if it is expressed in *minutes;*

$\Delta_0\, h$, the change of altitude in 1^m from the meridian passage due solely to the diurnal rotation (from Table 26);

$\Delta\, h$, the reduction of the maximum altitude; both expressed in *seconds.*

Now in the time t

$t\,\Delta\, d$ will be the excess of altitude produced by the change of declination and latitude;

$t^2\,\Delta_0\, h$ (as will be shown in Art. 199), the diminution of altitude due to diurnal rotation;

and we shall have

$$\Delta\, h = t\,\Delta\, d - t^2\,\Delta_0\, h.$$

But at a point whose hour-angle is $2\,t$, the altitude will be the same as the meridian altitude, or

$$0 = 2\,t\,\Delta\, d - (2\,t)^2\,\Delta_0\, h;$$

whence
$$t = \frac{\Delta\, d}{2\,\Delta_0\, h},$$

and
$$\Delta\, h = \tfrac{1}{2}\,t\,\Delta\, d = \frac{(\Delta\, d)^2}{4\,\Delta_0\, h} \qquad (94)$$

EXAMPLE: A ship in lat. 60 N., on March 21, sails south 14 miles per hour.

Observers change in latitude 14′ S. per hour or 14″ per minute.
Change in sun's declination 1′.0 per hour or 1″ per minute.

* Their sum, if they both tend to elevate or both to depress; otherwise their difference.

therefore $\triangle d = 14' + 1' = 15'$ per hour, or $15''$ per minute;
With Lat. 60 N. and Dec. 0° 23′ N. (Tab. 26).

gives $\triangle_0 h = 1''.1$;

$$t = \frac{\triangle d}{2\triangle_0 h} = \frac{15''}{2 \times 1''.1} = \frac{15''}{2.''2} = 6^m.81 \text{ or } 6^m\ 49^s$$

and $$\triangle h = \frac{15}{4.4} \times 15'' = 51.''$$

The uncertainty of altitudes at sea makes such a correction of little practical importance; but it is generally neglected by those navigators who work out their latitudes to seconds, supposing that they have attained that degree of accuracy. In the above example, the maximum altitude of the sun would have been greater than the meridian altitude, and the latitude obtained from it in error, by nearly 1′. The sun would not have sensibly *dipped* until 9 or 10 minutes after noon.

191. A difficulty occurs at sea in measuring the meridian altitude of the sun when it passes near the zenith, on account of its very rapid change of azimuth; the change being made from east to west, 180°, in a very few minutes.

What is wanted is the angular distance of the sun from the N. or S. points of the horizon. One of these points may be sufficiently fixed by means of the compass, and then the angular distance from this point observed within 1^m or 2^m of the meridian passage as determined by a watch regulated to apparent time.

192. From (92) we have $$z = L - d, \qquad (95)$$

by which the zenith distance may be found when the latitude and declination are given.

Also $d = L - z$, which may be used at sea for estimating the declination of a bright star from its estimated meridian altitude. If the time when it is near the meridian be also noted, and converted into sidereal time, we have the right ascension and declination of the star sufficiently near for determining what star it is.

EXAMPLE: July 16, 1918, in Lat. 11° N., at L. M. T., $8^h\ 45^m$, P.M., a bright star is seen near the meridian bearing S., at an approximate altitude of 55°. What star is it?

	h	m	s	
L. M. T., July 16	8	45	00	$L = 11°$ N.
R. A. M. S.	7	34	03	$z = 35$ N.
L. S. T.	16	19	03	$d = 24$ S.

The R. A. of Antares (α Scorpii) is $16^h\ 24^m$, and its declination 26° 15′ S. Therefore the star is Antares.

193. ***To find the latitude from an altitude of a heavenly body observed at any time, the local time of the observation and the longitude of the place being given.***

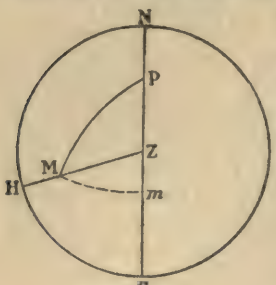


FIGURE 39

First Solution. Reduce the observed altitude to the true altitude, and from the local time and longitude find the declination and hour-angle of the body. (Arts. 135, 145, 146.) Then in the triangle P Z M (Fig. 39) there are given

$$Z\,P\,M = t,$$
$$P\,M = 90^\circ - d,$$
$$Z\,M = 90^\circ - h,$$

to find

$$P\,Z = 90^\circ - L.$$

By spherical trigonometry, if in the triangle A B C (Fig. 40) are given a, b, and A, we find c by the formulas

$$\left.\begin{aligned} \tan\phi &= \tan b \cos A, \\ \cos\phi' &= \frac{\cos\phi \cos a}{\cos b}, \\ c &= \phi \pm \phi'; \end{aligned}\right\}$$

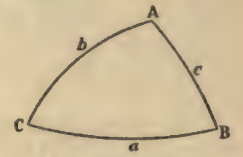


FIGURE 40

which, applied to the triangle P Z M, give

$$\left.\begin{aligned} \tan\phi &= \cot d \cos t, \\ \cos\phi' &= \frac{\cos\phi \sin h}{\sin d}, \\ 90^\circ - L &= \phi \pm \phi'. \end{aligned}\right\} \qquad (96)$$

These may be changed into a more convenient form for practice, if we put $\phi = 90^\circ - \phi''$; then

$$\left.\begin{aligned} \tan\phi'' &= \tan d \sec t, \\ \cos\phi' &= \frac{\sin\phi'' \sin h}{\sin d}, \\ L &= \phi'' \pm \phi'. \end{aligned}\right\} \qquad (97)$$

Here, observing that + and − may be rendered by N. and S. respectively, we mark ϕ'' N. or S. like the declination, and ϕ' either N. or S.; then the *sum* of ϕ'' and ϕ' when of the *same* name, their *difference* when of *different* names, is the latitude, of the same name as the greater. There are two values of L corresponding to the same altitude and hour-angle, but which, *unless* ϕ' *is very small*, will differ largely from each other; so that we may take that value which agrees best with the supposed latitude (at sea the latitude by account). When $t > 6^h$, $\phi'' > 90^\circ$, as in (69).

ϕ' is positive if $Z > 90°$, and negative if $Z < 90°$; the sign of ϕ' may therefore be determined by the bearing of the body.

194. In Fig. 40, if $M\,m$ be drawn perpendicular to the meridian, we shall have

$\phi = P\,m$, the polar distance of m,
$\phi'' = 90° - P\,m$, the declination "
$\phi' = Z\,m$, the zenith distance "

When ϕ' is very small (that is, when $M\,m$ nearly coincides with $M\,Z$), ϕ' cannot be found with precision from its cosine. If not greater than 12°, it can be found only to the nearest minute with 5-place tables; if only 2°, it can be found only within 3′. The more nearly, then, that $M\,m$ coincides with $Z\,m$, or, in other words, the nearer the body is to the prime vertical, the less accurate is the determination of the latitude. If the body is on the prime vertical, ϕ' cannot be found within 30′.

195. To find the effect of an error in the altitude, differentiate the second equation of (97), regarding ϕ' and h as variables; and we have

$$d\phi' = -\frac{\sin\phi'' \cos h}{\sin d \sin\phi'} d h$$

or, since

$$\frac{\cos\phi'}{\sin h} = \frac{\sin\phi''}{\sin d},$$

$$d\phi' = -d h.\ \cot\phi' \cot h. \qquad (98)$$

But in the triangle $M\,Z\,m$,

$$\cos M\,Z\,m = -\cos M\,Z\,P = \frac{\tan m\,Z}{\tan M\,Z};$$

that is, Z being the azimuth,

$$-\cos Z = \frac{\tan\phi'}{\cot h}, \text{ or } \sec Z = -\cot\phi' \cot h, \qquad (99)$$

and therefore,

$$d\phi' = d h \sec Z. \qquad (100)$$

If the body is on the meridian, $Z = 0$ or 180°, and numerically $d\phi' = d h$.

The nearer Z is to 90°, the greater is $d\phi'$. If $Z = 90°$, or the body is on the prime vertical, $\sec Z = \infty$, and $d\phi'$ is incalculable. If Z is near 90°, (97) is inaccurate.

A star which transits the meridian near the zenith, changes its azimuth very rapidly. Unless observed on the meridian, it cannot be depended on for latitude.

196. From (97) we see that should the body be in the equator, the formula cannot be used because the cosec d becomes infinity. A good latitude may, however, be obtained when the body is in the equator by substituting for formula (97) the following equivalent group. The auxiliary quantity α is the length of the perpendicular let fall from the star upon the meridian; thus:

$$\begin{aligned} \sin \alpha &= \cos d \sin t \\ \sin \phi'' &= \sin d \sec \alpha \\ \cos \phi' &= \sin h \sec \alpha \end{aligned}$$

These equations retain their finite form no matter what the value of d.

When $d = 0$; $\phi'' = 0$ and $\cos \phi' = \cos \phi = \sin h \sec t$ (101). This formula is then used as a special case of the $\phi'' \phi'$ sight.

197. To find the effect of an error in the time, and consequently in the hour-angle, we may take the formula (71):

$$\sin h = \sin L \sin d + \cos L \cos d \cos t,$$

and differentiate regarding L and t as variables; which gives,

$$\mathrm{d}L = -\frac{\cos L \cos d \sin t}{\sin L \cos d \cos t - \cos L \sin d,} \cdot \mathrm{d}t.$$

But, $\cos d \sin t = \cos h \sin Z,$
and $\sin L \cos d \cos t - \cos L \sin d = \cos h \cos Z$ } Sph. Trig.

so that, $$\mathrm{d}L = -15\, \mathrm{d}t \cos L \tan Z, \qquad (102)$$

which requires that the azimuth should be known.

At sea the chief uncertainty of this problem is in the time, either from its imperfect determination by observation, or from unavoidable errors in allowing for the run of the ship in the interval between the observations for time and for latitude.

By (102) it appears that the effect of an error in the time is 0 when $Z = 0$ or 180°, that is, when the body is on the meridian; and the effect is incalculable when $Z = 90°$ or 270°, or the body is on the prime vertical.

Moreover, the effect is opposite on different sides of the meridian, and would be eliminated by two observations of the same body, or of different bodies, at the same azimuth E. and W. of the meridian.

198. *Second Solution.* If the latitude is already approximately known, we have (71)

$$\sin h = \sin L \sin d + \cos L \cos d \cos t;$$

whence $$\cos (L - d) = \sin h + 2 \cos L \cos d \sin^2 \tfrac{1}{2} t;$$

or since $(L - d)$ is the meridian zenith distance of the body, (92), denoting it by z_0, and the meridian altitude by h_0, we have

$$\cos z_0 = \sin h_0 = \sin h + 2 \cos L \cos d \sin^2 \tfrac{1}{2} t, \qquad (103)$$

in which we may use the approximate value of L in computing the term $2 \cos L \cos d \sin^2 \frac{1}{2} t$, which term is smaller the nearer the observation is taken to the meridian. Having found the meridian zenith distance, we may find the latitudes as in Art. 188. If the computed value of L differs largely from the assumed value, the computation should be repeated, using this new value.

199. *Third Solution. Reduction to the Meridian.* When the observation is taken very near the meridian, we may find the correction to be applied to the observed altitude to reduce it to the meridian altitude, thus:

From (103) we have

$$\sin h_0 - \sin h = 2 \cos L \cos d \sin^2 \tfrac{1}{2} t,$$

whence, by plane trigonometry,

$$\cos \tfrac{1}{2} (h_0 + h) \sin \tfrac{1}{2} (h_0 - h) = \cos L \cos d \sin^2 \tfrac{1}{2} t.$$

But h_0 and h differing very little, we may put

$$\cos \tfrac{1}{2} (h_0 + h) = \cos h_0 = \sin z_0 = \sin (L - d),$$

so that

$$\sin \tfrac{1}{2} (h_0 - h) = \frac{\cos L \cos d \sin^2 \frac{1}{2} t}{\sin (L - d)}. \qquad (104)$$

Put $\Delta h = h_0 - h$, the reduction of the observed to the meridian altitude, or, as it is usually called, *The reduction to the meridian;* and, since Δh and t are quite small, put

$\sin \frac{1}{2} \Delta h = \frac{1}{2} \Delta h \sin 1''$ (Δh being expressed in seconds of arc),

$\sin \frac{1}{2} t = \frac{1}{2} t \times 15 \sin 1''$ (t " " " of time),

then (104) reduces to

$$\Delta h = \frac{112.5 \sin 1'' \cos L \cos d}{\sin (L - d)} \times t^2;$$

or, since $\sin 1'' = 0.000004848$,

$$\Delta h = \frac{0''.000545 \cos L \cos d}{\sin (L - d)} \times t^2 \ (t \text{ in seconds}).$$

In this formula t is in seconds of time; but if, as is usual, t is expressed in *minutes*, we must put $(60\ t)^2$ for t^2, so that we have

$$\Delta h = \frac{1''.96349 \cos L \cos d}{\sin (L - d)} \times t^2. \qquad (105)$$

If $t = 1^m$, the formula expresses the change of altitude in one minute from the meridian. Representing this by $\Delta_0 h$, we have

$$\Delta_0 h = \frac{1''.96349 \cos L \cos d}{\sin (L - d)} \qquad (106)$$

and

$$\left.\begin{aligned} \Delta h &= t^2 \Delta_0 h, \\ h_0 &= h + \Delta h, \text{ the meridian altitude.} \end{aligned}\right\} \qquad (107)$$

Whence the latitude is found as by a meridian altitude (Art. 188). Art. 326 (Bowditch).

Bowditch's Table 26 contains the values of $\Delta_0 h$ for each 1° of declination from 0 to 63°, and each 1° of latitude from 0 to 60°; except when $L - d < 4°$, for then Δh is so large that (105) and (106) become inaccurate. In this case the body is near the zenith, and altitudes out of the meridian do not afford a reliable determination of the latitude.

Bowditch's Table 27 contains t^2 for each half minute between $0^m\ 30^s$ and $26^m\ 0^s$, and for all variations likely to be used in the method of "reduction to the meridian."

When h is small, the reduction to the meridian may be found by this method quite accurately, for the greater the value of $\Delta_0 h$, the smaller the limit of t; so for a value of $\Delta_0 h = 2''.0$, Δh is found for t, as great as 26^m. If h is near 90°, t must be taken within much narrower limits, for instance for a value of $\Delta_0 h = 28''$ it is found for values of t of and less than 5^m. Again where h is near 90°, z_0, or its equal $(L - d)$, is very small, and consequently $\Delta_0 h$ becomes large. Unless then t is sufficiently small, Δh will be too great for the assumption $\sin \frac{1}{2} \Delta h = \Delta h \sin 1''$.

If $d > L$, $\sin (L - d) = \sin z_0$ is negative; that is, z_0 will have a different name or sign from L (Art. 188). Properly h, h_0, and $\Delta_0 h$ would also become negative to correspond. Still, however, we shall have numerically

$$h_0 = h + \Delta h.$$

We may therefore disregard the sign of $L - d$ in (106), and consider h and h_0 as always positive.

If the star is observed at its lower culmination, then t will be the hour-angle from the lower branch of the meridian, and for d we may use $180° - d$ (Art. 188). $\Delta_0 h$ and Δh are then numerically subtractive.

Limit of Hour-angle. — The hour angle of the sun or the time from noon in minutes should not be greater than the number of degrees in the meridian zenith distance of the sun. A good rule to follow then is always, find latitude by a $\phi''\ \phi'$ sight when the time from noon is greater than the limits of Table 27.

EXAMPLE 1: July 17, 1918, in Lat. 36° 38′ S., Lon. 105° 18′ E., by D. R. the sextant altitude of the sun's lower limb bearing northward and westward was 30° 15′ 00″, I. C. + 2′ 30″, height of eye 17 feet, watch time of observation $1^h\ 00^m\ 00^s$ P.M., C–W $4^h\ 47^m\ 14^s$, chronometer slow $12^m\ 46^s$. Find the latitude ($\phi''\ \phi'$ sight).

Time				Altitude				Eq. of Time	Declination
	h	*m*	*s*		°	′	″	At 18^h $-5^m\ 52^s.3$	At 18^h N. 21° 20′.7
W.	1	00	00	Obs.	30	15	00		
C – W	4	47	14	Corr.	+	12	43		
C	5	47	14	*h* =	30	27	43		
c. c.	+	12	46	Corr. (Tab. 46 Bowd.) =		+10	13		
G. M. T. July 16	18	00	00	I. C. =		+ 2	30		
Eq. of t.		− 5	52.3	Corr.		+12	43		
G. A. T.	17	54	07.7						
Lon. E.	7	01	12.0						
L. A. T.	0	55	19.7						
	t = 13°	49′	56″ W.						

h =	30	27	43		sin 9.70497
*t** =	13	49	56	sec 0.01278	
d =	21	20	42 N.	tan 9.59194	cosec 0.43893
ϕ'' =	21	55	21 N.	tan 9.60472	sin 9.57212
ϕ' =	58	40	00 S.		cos 9.71602
Lat. =	36	44	39 S.		

If we suppose an uncertainty of 3′ in the altitude and 20′ in the longitude, by (99), (100), and (102)

	l. cot (−*h*) − 0.2305		l. cos *L* . 9.904	
	l. cot ϕ' 9.7845	−d *t* = −20′	log −1.301	
Z = S. 165° 02′ W.	l. sec *Z* −0.0150		l. tan *Z* − 9.427	
d *h* = +3′	log 0.477	−d *L* = +4′.2	log 0.632	
d *L* = −3′.1	log −0.493			

* Instead of changing *t* into arc, we may enter col. P.M. of Table 44 with $2t = 1^h\ 50^m\ 39.4^s$.

That is, an increase of $3'$ in the altitude will numerically decrease the latitude $3'.1$; and a numerical increase of $20'$ in the assumed longitude will increase the latitude $4'.2$. This may be conveniently expressed in the following way:

$$\text{Lon.} \quad 105^\circ\ 18' \doteq 20' \quad \text{E.};\ \odot,\ 30^\circ\ 15' \doteq 3'$$
$$L = \ 36^\circ\ 44'.6 \doteq 4'.2 \mp 3'.1 \text{ S.}$$

EXAMPLE 2: December 16, 1918, in Lat. 55° 30′ 00″ N., Lon. 135° 25′ 00″ W., by D. R. the sextant altitude of the planet Jupiter bearing S^d. and W$^d_.$ was 40° 01′ 30″, I. C. − 3′ 00″ height of eye 19 feet, W. T. = $4^h\ 48^m\ 45^s$ A.M., C–W $9^h\ 12^m\ 07^s$, chronometer fast $12^m\ 34^s$. Find the latitude ($\phi''\ \phi'$).

Time				Alt.			Alt. Corr.			R. A.				Dec.			
	°	′	″	°	′	″		′	″	h	m	s	H. D.	°	′	H. D.
W.	4	48	45	Obs.	40	01	30	I. C.	−3	00							
C − W	9	12	07	Corr.	−	8	25	Dip.	−4	16	Dec. 16 =	6	56	19	−1^s.29	Dec. 16	+22	48.7	+0′.033
C.	2	00	52	True	39	53	05	r	−1	09	Corr.			−2.3	G. M. T. $1^h.8$	Corr.	+	1	G. M. T. $1^h.8$
c. c.	−	12	34					Corr.	−8	25		6	56	16.7	Corr. −2^s.32	Dec.	N. 22	48.8	Corr. +0′.059
G. M. T.	1	48	18					or											
Dec. 16		=	$1^h.8$					(Tab. 46 Bowd.) =	−5	25						R. A. M. S.			
	h	m	s					I. C.	−3	00									
G. M. T.	1	48	18					Corr.	−8	25							h	m	s
R. A. M. S.	17	37	34.1													G. M. N.	17	37	16.2
G. S. T.	19	25	52.1													Corr. G. M. T.		+0	17.9
λ W.	9	01	40.0													R. A. M. S.	17	37	34.1
L. S. T.	10	24	12.1																
*☉ R. A.	6	56	16.7																
t =	3	27	55.4																
=	51°	58′	51″																

	°	′	″			
h =	39	53	05		sin	9.80702
t =	51	58	51	sec 0.21047		
d =	22	48	48 N.	tan 9.62391	cosec	0.41147
ϕ'' =	34	19	51 N.	tan 9.83438	sin	9.75126
ϕ'	21	08	12 N.		cos	9.96975
Lat. =	55	28	03 N.			

EXAMPLE 3: March 21, 1918, in Lat. 66° 30′ N., Lon. 1° W., by D. R. the sextant altitude of the sun's lower limb bearing S^d. and E^d. was 20° 49′ 00″, I. C. − 3′ 00″, height of eye 19 feet, W. T. of observation 10^h 14^m 00^s A.M., C–W 0^h 13^m 23^s, chronometer fast 3^m 33^s. Find the latitude (ϕ'' ϕ' special case when body is on the equator, Form. 101).

Here $\phi'' = 0$, and $\cos \phi' = \cos \phi = \sin h \sec t$

therefore $\cos L = \sin h \sec t$

Time

	h	m	s
W.	10	14	00
C − W	0	13	33
C	10	27	33
c. c.	−	3	33
G. M. T. Mar. 20th	22	24	00
	= 22^h.4		

	h	m	s
G. M. T.	22	24	00
Eq. t.	−	7	29.2
G. A. T.	22	16	30.8
Lon. W.	0	4	00.0
L. A. T.	22	12	30.8
t =	1	47	29.2
t =	26°	52′	18″

Alt.

	°	′	″
Obs. *h*	20	49	00
Corr.		+6	25
True *h*	20	55	25

Alt. Corr.

	′	″
S. D.	+16	05
I. C.	− 3	00
Dip.	− 4	16
p & *r*	− 2	24
Corr.	+ 6	25

Dec. and Eq. of T.

	°	′	
Dec. 22^h	−0	00.4	*H. D.* −1′.0
Corr.		−0.4	G. M. T. .4
Dec.	0	00.0	Corr. −0′.4

	m	s	
Eq. t. 22^h	−7	29.5	*H. D.* −0^s.7
Corr.	−	.3	G. M. T. 0^h.4
Eq. t.	−7	29.2	Corr. −0^s.28

	°	′	″		
h =	20	55	25	sin	9.55282
t =	26	52	18	sec	0.04963
Lat. =	66	24	00 N.	cos	9.60245

EXAMPLE 4: May 15, 1918, in Lat. 41° 22′ N., Lon. 71° 52′ 30″ W., by D. R. the sextant altitude of the sun's lower limb bearing southward was 67° 18′ 00″, I. C. − 1′ 50″, height of eye 25 feet, watch time of observation $12^h\ 04^m\ 00^s$ P.M., C–W $5^h\ 34^m\ 28^s$, chronometer fast $51^m\ 48^s.5$. Find the latitude.

Time				Alt.				Corr. Alt.			Dec. and Eq. of T.		
	h	m	s		°	′	″		′	″		°	′
W.	12	04	00	Obs. h	67	18	00	S. D. =	+15	51	Dec. 4^h =	+18	46.7
C − W.	5	34	28	Corr.	+	8	46	I. C. =	− 1	50	Corr.	+	0.5
C.	5	38	28	h =	67	26	46	Dip. =	− 4	54	Dec.	N. 18	47.2
c. c.	−	51	48.5	$\Delta_0 ht^2$ =		+0	31	p & r =	− 0	21			
G. M. T., May 15	4	46	39.5	h	67	27	17 S.	Corr.	+ 8	46			
	= $4^h.78$			z =	22	32	43 N.						
	h	m	s	d =	18	47	12 N.						
G. M. T. =	4	46	39.5	Lat. =	41	19	55 N.						
Eq. t.	+	3	48.5										
G. A. T.	4	50	28										
Lon. W.	4	47	30										
L. A. T.	0	02	58										

$t = 0^h\ 02^m\ 58^s$
$\Delta_0 h = 3''.6$ (Tab. 26 Bowd.)
$\Delta_0 ht^2 = 0'\ 31''$ (Tab. 27)

		H. D.
Dec. 4^h =	+18 46.7	+0′.6
Corr.	+ 0.5	G. M. T. $0^h.78$
Dec.	N. 18 47.2	Corr. +0′.468

Eq. t. = $+3^m\ 48^s.5$

200. ***To find the latitude from a number of altitudes observed very near the meridian, the local times being known.***

Solution. By (107) we see that very near the meridian the altitude of a body varies very nearly as the square of its hour-angle. Hence we cannot regard the mean of several altitudes as cor-

responding to the mean of the times, since this is assuming that the altitude varies as the hour-angle. Let,

h_1, h_2, h_3, etc., be the several altitudes;

t_1, t_2, t_3, etc., the corresponding hour-angles expressed in *minutes;*

and we have as the reduction of each altitude to the meridian, and the deduced meridian altitude,

$$\left.\begin{array}{ll} \Delta_1 h = t_1^2 \,.\, \Delta_0 h & h_0 = h_1 + \Delta_1 h \\ \Delta_2 h = t_2^2 \,.\, \Delta_0 h & h_0 = h_2 + \Delta_2 h \\ \Delta_3 h = t_3^2 \,.\, \Delta_0 h \text{ etc.} & h_0 = h_3 + \Delta_3 h \end{array}\right\} \text{etc.} \qquad (108)$$

Thus the meridian altitude may be derived from each altitude, and the mean of all these meridian altitudes taken as the correct meridian altitude. But the following is a more expeditious method:

If n is the number of observations, the mean value of h_0 will be

$$h_0 = \frac{h_1 + h_2 + h_3 + \ldots h_n}{n} + \frac{\Delta_1 h + \Delta_2 h + \Delta_3 h + \ldots \Delta_n h}{n}$$

or,

$$h_0 = \frac{h_1 + h_2 + h_3 + \ldots h_n}{n} + \frac{t_1^2 + t_2^2 + t_3^2 + \ldots t_n^2}{n} \Delta_0 h \qquad (109)$$

Whence the rule:

Take the mean of the squares of the hour-angles in *minutes* (Table 27, Bowditch); multiply it by the change of altitude in 1^m from the meridian (Table 26); and add the product to the mean of the altitudes. The result is the mean meridian altitude required. (Bowditch, Art. 326.) From the meridian altitude thus found, deduce the latitude as from any other meridian altitude. (Art. 188.) Strictly, however, the declination to be used is that which corresponds to the mean of the times, and the hour-angles, t, are intervals of *apparent* time for the sun, and of *sidereal* time for a fixed star.

201. It is unnecessary to reduce each observed altitude separately to a true altitude; as the reductions, excepting slight changes of refraction and parallax, are the same for all, and may be computed for the mean of the observed altitudes, and applied to this mean with the reduction to the meridian.

202. Should it be desirable to compare the several observations with each other, and test their agreement, it will be sufficient to compute the several reductions to the meridian, $\Delta_1 h$, $\Delta_2 h$, $\Delta_3 h$, etc., and apply them separately to the *readings* of the instrument; or to the *half-readings* when the altitudes are observed with an artificial horizon: applying, also, the semidiameter when both limbs of the body are observed.

203. If the altitudes are taken on both sides of the meridian, and at nearly corresponding intervals, a small error in the local time will but slightly affect the result; for such error will make the estimated hour-angles and the corresponding reductions on one side of the meridian too large, and on the other side too small.

204. This method is rarely used at sea, as a single altitude on or near the meridian suffices. No increase of the number of observations will diminish at all the error of the dip, which affects alike each observation and the mean of all.* But on land it is preferable to measure a number of altitudes at the same culmination of the body, and thus diminish the "error of observation." Altitudes of the sun are used, but the best determinations are from the altitudes of a bright star. To facilitate the operations, and avoid mistaking one star for another, it is well to compute the altitude approximately beforehand. (Art. 192.)

If an artificial horizon is employed, the error of the roof is partially eliminated by making two sets of observations with the roof in reversed positions.

205. If two stars are observed which culminate at nearly the same altitude, one north, the other south of the zenith, the error of the instrument is nearly eliminated; for such error (except *accidental* error of graduation) will make the latitude from one of the stars too great, and that from the other too small by very nearly the same amount; the more nearly, the less the difference of the altitudes. The error peculiar to the observer is also eliminated.

If the observations are made with an artificial horizon, the error of the roof is eliminated if the *same* end is toward the observer in both sets of observations.

206. Bowditch's Table 26 gives the variation of the altitude of any heavenly body, for one minute of time from meridian passage, for latitudes up to 60°, declinations to 63°, and altitudes between 6° and 86°. Altitudes below 6° are unreliable on account of refraction, above 86° the method is inapplicable because the body transits so near the zenith. If a star is used whose declination is beyond this limit, or if greater precision than the table affords is required, $\Delta_0 h$ may be computed for the star and place by (106).

$$\Delta_0 h = \frac{1''.9635 \cos L \cos d}{\sin (L - d)}$$

* Such an error is called *constant;* those which affect the several observations differently are called *variable.*

207. If the observations are made at the lower culmination of the star, we have only to use in the formulas $180° - d$ instead of d. (Art. 188.)

The altitudes observed at the same culmination are very nearly the same. To render the measurements independent, after each observation move slightly the tangent screw of the instrument. With the sextant, it is best to make the final motion of the tangent screw at each observation always in the same direction; for example, in advance.

EXAMPLE 1: May 25, 1918, P.M. at St. George Island, Florida, Lat. 29° 36′ N., Lon. 85° 03′ W., observed the following circummeridian altitudes of the star Spica (α Virginis) bearing S. for latitude using a sextant and artificial horizon I. C. $- 0' 10''$, watch times as noted below, C − W $5^h\ 30^m\ 41^s.9$, chronometer fast on G. M. T. $49^m\ 59^s.1$. Find the latitude.

Watch Time of Transit				W. T. of Obs.			Mean Time int. t.		Sidereal Time int. t.		β	Double Alt.			Mer Alt. and Lat.				Star R. A. and Dec.
	h	m	s	h	m	s	m	s	m	s	m	°	′	″		°	′	″	
May 25, R. A. M. S.	4	09	02.3	8	59	38	−10	42	−10	44	115.2	99	11	50	* s 2h,	99	18	26	R. A. 13 h 20 m 55.5
Red. λ (Tab. III)	+	0	55.9	9	02	06	− 8	14	− 8	15	68.1	99	15	50	I. C.		−	10	Dec. 10° 44′.3 S.
Sid. time local 0h	4	09	58.2	9	04	18	− 6	02	− 6	03	36.6	99	19	00		2	99	18	16	
R. A. Spica = L. S. T.	13	20	55.5	9	05	38	− 4	42	− 4	43	22.2	99	21	40	*'s h'		49	39	08	
Sid. int.	9	10	57.3	9	07	50	− 2	30	− 2	30	6.2	99	21	40	Ref.			−	49	
Red. (Tab. II)		− 1	30.3	9	09	40	− 0	40	− 0	40	0.4	99	24	00	*'s h1		49	38	19	
L. M. T. of transit	9	09	27.0	9	13	27	+ 3	07	+ 3	07	9.7	99	23	30	△h			+2	09	
Lon. W.	5	40	12.0	9	15	15	+ 4	55	+ 4	56	24.3	99	20	10	*'s h0		49	40	28 S.	
G. M. T. of transit	14	49	39.0	9	19	03	+ 8	43	+ 8	44	76.3	99	14	50	*'s0		40	19	32 N.	
Chro. fast on G. M. T.	+	49	59.1	9	22	07	+11	47	+11	49	139.6	99	11	50	*'d		10	44	18 S.	
C. T. *'s transit	3	39	38.1						Mean		49.86	99	18	26	Lat.		29	35	14 N.	
C − W	5	30	41.9																	
W. T. of *'s transit	9	10	20																	

$1''.9635$
$L = 29° 36'$
$d = -10° 44'$
$L - d = 40° 20'$
$\triangle 0^h = 2''.59$
$\beta = 49.86$
$\triangle h = 2' 09''$

log	0.2930
cos	9.9393
cos	9.9923
cosec	0.1889
log	0.4135
log	1.6978
log	2.1113 = 129″

208. ***To find a constant for circummeridian altitudes of the sun near the upper transit*** — Find the difference in longitude from noon of the day before until noon of sight by taking the run until 11 A.M. from the log and estimating the run from there until noon. This difference of longitude changed to time gives the amount the deck watch must be set back or ahead to be correct at the meridian.

Find the longitude at L. A. noon from the A.M. longitude and the run of vessel and with that longitude find watch time of noon.

EXAMPLE 1: March 4, 1918, in Lat. 20° 00′ 30″ S., Lon. 29° 40′ 42″ W., by D. R. at $8^h\ 00^m\ 00^s$ A.M. a time sight gave the watch as fast $4^m\ 30^s$ on L. A. T. The vessel steamed till apparent noon 150° (true) 10 knots; it was estimated that the vessels' clocks would be set ahead about 8 minutes. Find the longitude at noon by D. R.; W. T. of noon; constant for meridian altitude; I. C. 0.; height of eye 32 feet; also find constant to give noon latitude, when applied to sextant altitudes observed at 14^m, 10^m, 7^m, 4^m, 2^m and also W. T. of first observation and of noon. Find the latitude.

Clock set ahead 8 minutes, the run to noon from 8 A.M. will be $10 \times 3^h.87 = 38.7$ miles.

	True course	Dist.	D. L.	Dep.	D. Lo.
	150°	38.7	33.5 S.	19.4 E.	20′.7 E.

	°	′	″		°	′	″	
Lat. 8 A.M.	20	00	30	S.	Lon. 8 A.M.	29	40	42 W.
Diff. Lat.		33	30	S.	Diff. Lon.		20	42 E.
Lat. noon, D. R.	20	34	00		Lon. noon, D. R.	29	20	00 W.

	h	m	s
G. A. T. of L. A. noon	1	57	20
Eq. of T.	+	11	58
G. M. T. of L. A. noon	2	09	18
	= $2^h.155$		

To find W. T. of noon

	m	s
$8^h\ 00^m\ 00^s$ A.M., watch fast on L. A. T.	4	30
Change in time for diff. lon. 20′.7 (slow)	1	22.8
Watch fast at noon on L. A. T.	3	07.2

W. T. of L. A. N. = $12^h\ 03^m\ 07^s.2$

W. T. of first ex. meridian sight $11^h\ 49^m\ 07^s.2$

The better way is to set the watch back $3^m\ 07^s$ and have it correct at noon.

To find the approximate altitude

		°	′	″
Dec.-March 4, 2^h	S.	6	36.5	
Corr. $1' \times 0^h.155$			−.2	
Dec. at L. A. N.	S.	6	36.3	
$90° + d$		96	36	18
Approx. corr.			−10	37
$90° + d - c$		96	25	41
Lat. (approx.)	S.	20	39	00
Alt. (approx.)		75	46	41

To find the constant

	′	″
I. C.	0	00
S. D.	+16	10
Dip	−5	33
App. corr.	+10	37
p & r	−	13
Cor.	+10	24

	°	′	″
Dec. at L. A. N.	6	36	18
$90° + d$	96	36	18
Corr.		−10	24
Constant	96	25	54

		Interval from noon	Tab. 27	△ L	Error	Constant	Altitudes	Latitude
			′ ″	′ ″	′ ″	° ′ ″	° ′ ″	° ′ ″
True course, 150°	Dist. 10	D. L. 8.7	14 min. −24 30	+2 02	−22 28	96 03 26	75 30 00	20 33 26 S.
		10 " −12 30	+1 27	−11 03	96 14 51	75 41 00	20 33 51 S.
		7 " − 6 07	+1 01	− 5 06	96 20 48	75 47 00	20 33 48 S.
(Tab. 26 Bowd.) $\Delta h = 7''.5$		4 " − 2 00	+0 35	− 1 25	96 24 29	75 50 00	20 34 29 S.
On above course D. L. per hour = 8′.7		2 " − 0 30	+0 17	− 0 13	96 25 41	75 51 00	20 34 41 S.
or D. L. per min. 8″.7						Mean	20 34 03 S. = latitude.	

209. ***To find the latitude from two altitudes near the meridian when the time is not known.*** **(*Chauvenet's Method*).***

The method of reducing circummeridian altitudes to the meridian, when the time is known, has already been given (Art. 200). At sea, however, the local time is frequently uncertain, while altitudes near the meridian are resorted to as next in importance to meridian altitudes for finding the latitude.

As in Art. 200, let h_0 represent the meridian altitude,

$\Delta_0 h = \dfrac{1''.96349 \cos L \cos d}{\sin (L - d)}$, the change of altitude in 1^m from the meridian (Table 26, Bowditch), and as before,

h and h', the true altitudes,
T and T', the corresponding hour-angles (in minutes of time),
$t = T' - T$, the difference of the hour-angles,
$T_0 = \frac{1}{2}(T' + T)$, the middle hour-angle.
By (105),

$$\left.\begin{aligned} h_0 &= h + \Delta_0 h\, T^2, \\ h_0 &= h' + \Delta_0 h\, T'^2. \end{aligned}\right\} \tag{110}$$

The mean of these equations is

$$h_0 = \tfrac{1}{2}(h + h') + \tfrac{1}{2}(T'^2 + T^2)\,\Delta_0 h, \tag{111}$$

But

$$\tfrac{1}{2}(T'^2 + T^2) = \left(\frac{T' - T}{2}\right)^2 + \left(\frac{T' + T}{2}\right)^2 = (\tfrac{1}{2}t)^2 + T_0^2,$$

which, substituted in (111), gives

$$h_0 = \tfrac{1}{2}(h + h') + \left[\tfrac{1}{4}t^2 + T_0^2\right]\Delta_0 h. \tag{112}$$

The difference of the two equations of (110) gives

$$h - h' = (T'^2 - T^2)\,\Delta_0 h = 2\,T_0 t\,\Delta_0 h.$$

Hence,
$$T_0 = \frac{\frac{1}{2}(h - h')}{t\,\Delta_0 h} = \frac{\frac{1}{4}(h - h')}{\frac{1}{2}t\,\Delta_0 h}. \tag{113}$$

Substituting this in (112), we have

$$h_0 = \tfrac{1}{2}(h + h') + (\tfrac{1}{2}t)^2\,\Delta_0 h + \frac{\left[\frac{1}{4}(h - h')\right]^2}{(\frac{1}{2}t)^2\,\Delta_0 h}. \tag{114}$$

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The reduction to the meridian, then, is effected "by adding to the mean of the two altitudes two corrections; 1st, the quantity $(\frac{1}{2}\,t)^2\,\Delta_0\,h$, which is nothing more than the *common reduction to the meridian* (107), computed with the half-elapsed time as the hour-angle; 2d, the square of one-fourththe difference of the altitudes divided by the first correction." Several pairs of altitudes can be thus combined, and the mean of the meridian altitudes taken, from which the latitude can be obtained as from an observed meridian altitude.

210. The restriction of the method corresponds with that of *reduction to the meridian* (Art. 199).* Quite accurate results can be obtained with hour-angles limited to 5^m when the altitude is 80°, to 25^m when the altitude is only 10°. If the interval t, however, exceed 10^m, $\Delta_0\,h$ should be computed to two or three places of decimals, as it is given in Table 26 (Bowditch) only to the nearest $0''.1$.

The accuracy of the method depends mainly upon the accuracy of the second correction, and therefore upon the precision with which the difference of altitudes has been obtained. The altitudes, then, should be observed with great care. Errors of the tabulated dip and refraction, and a constant error of the instrument, will affect both altitudes nearly alike. If the altitudes are equal, this second correction becomes 0. The most favorable condition is, therefore, that of equal altitudes observed on each side of the meridian.

At sea, the method is especially useful for altitudes of the sun observed with a clear, distinct horizon. A long interval between the observations is to be avoided on account of the uncertainty of the reduction of one of the altitudes for the run of the vessel.

211. The hour-angle of either altitude may also be obtained approximately; for we have from (113), in minutes,

$$T^0 = \frac{\frac{1}{4}\,(h - h')}{\frac{1}{2}\,t\,\Delta_0\,h}, \qquad \left.\begin{array}{l} \\ \\ \end{array}\right\} \quad (115)$$

and

$$T = T_0 - \tfrac{1}{2}\,t, \qquad T' = T_0 + \tfrac{1}{2}\,t. \quad \left.\begin{array}{l} \\ \\ \end{array}\right\}$$

* Table 26 (Bowd.) gives $\Delta_0\,h$ only to the nearest $0''.1$; if, then, it is taken from this table, $\Delta_0\,h\,t^2$ may be in error $1''$, if $t > 4^m$. If, however, $\Delta_0\,h$ is computed to the nearest $0''.001$, the error of using $\Delta_0\,h\,t^2$ will not exceed $1''$, unless $t > 20^m$ and $h > 60°$.

EXAMPLE: September 3, 1918, near noon, in Lat. 37° 40′ N., Lon. 75° 00′ W. observed with a sextant two altitudes of the sun's lower limb bearing southward, the first at $12^h\ 30^m\ 21^s$ P.M., altitude 59° 43′ 40″, the second at $12^h\ 35^m\ 36^s$ P.M., altitude 59° 38′ 40″, I. C. − 0′ 30″, height of eye 18 feet. Find the latitude.

Time and $\frac{t}{2}$

	h	m	s
W.	12	30	21
W.′	12	35	36
t =		5	15
$\frac{t}{2}$ =		2	37.5

$\triangle_0 h$ (Tab. 26) 3″.09

$\left(\frac{t}{2}\right)^2$ $\triangle_0 h$ = 0′ 21″.6

(Tab. 27) = 1st corr.

Difference of Obs. Altitudes

	°	′	″
h	59	43	40
h^1	59	38	40
$h - h^1$ =		5	00
$\frac{h - h^1}{4}$ =		1	15

$\left(\frac{h - h^1}{4}\right)^2 = (75'')^2 = 5625''$

$\frac{h + h^1}{2}$ = 59° 41′ 10″

Mean of Altitudes and Latitude

	°	′	″	
$\frac{h + h^1}{2}$ =	59	41	10	
Corr.	+	10	45	
	59	51	55	
1st corr.	+		22	
2d corr.	+	4	20	
h_0	59	56	37	S.
z_0	30	03	23	N.
d	7	41	06	N.
Lat. =	37	44	29	N.

Corr. to Alt.

	°	″
S. D. =	+15	54
I. C.	− 0	30
Dip	− 4	09
p & r	− 0	30
Corr.	+10	45

G. A. T. of local app. noon

	h	m	s
Sept. 3	5	00	00
Eq. t.	−	0	33
G. M. T. of local app. noon = 5^h	4	59	27

	°	′ Dec.	
			H. D.
At 4^h	+7	42.0	−0.′0
Corr.	−	0.9	G. M. T. 1^h
Dec. = N.	7	41.1	Corr. −0′.9

$5625'' = \log\ 3.75012$

$21''.6 = \log\ \underline{1.33445}$

2d corr. = $260''.4 = \log\ 2.41567$

2d corr. = 4′ 20″.4

212. ***To find the latitude from an observed altitude of Polaris or the North Pole-star.***

Solution. The formulas (96) of Art. 193.

$$\tan \phi = \cot d \cos t$$

$$\cos \phi' = \frac{\cos \phi \sin h}{\sin d}$$

$$90^\circ - L = \phi = \phi'$$

can be greatly simplified in the case of the Pole-star, owing to the small polar distance.

Putting $d = 90^\circ - p$ and $\phi' = 90^\circ - \phi''$, we have

$$\left.\begin{aligned} \tan \phi &= \tan p \cos t \\ \text{or} \quad \phi &= p \cos t \text{ (within } 0''.5) \\ \sin \phi'' &= \sin h \frac{\cos \phi}{\cos p} \\ L &= \phi'' - \phi, \end{aligned}\right\} \qquad (116)$$

the 2d value of L, or $(180^\circ - \phi'' - \phi)$, being excluded, as it exceeds 90°. p and ϕ are so small, that the cosine of each is nearly 1, and consequently

$$\sin \phi'' = \sin h \qquad \text{and} \qquad \phi'' = h, \text{ nearly.}$$

Thus we have

$$\left.\begin{aligned} \phi &= p \cos t \\ L &= h - \phi \end{aligned}\right\} \qquad (117)$$

or $$L = h - p \cos t$$

If t is more than 6^h or less than 18^h, $\cos t$ is negative, and we have numerically

$$L = h + \phi,$$

or $$L = h + p \cos t.$$

Let S represent the sidereal time, and a the right ascension of the star, then

$$t = S - a \qquad \text{and} \qquad \phi = p \cos (S - a).$$

If we consider the right ascension and polar distance of the star to be constant, ϕ may be computed and tabulated for different hour-angles of the star, as in Table 4, Ephemeris, or for different L. S. T. as in the Almanac. Owing to the change of right ascension and declination, such a table requires correction for each year. It will furnish an approximate value of the latitude; but it is more accurate to take the *apparent*

right ascension and declination from the Almanac, and compute t and ϕ.

ϕ may be found approximately in the traverse table (Table 2) in the *Lat. col.*, by entering the table with t as a *course*, and p as a *distance*.

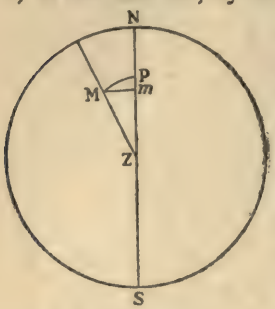

FIGURE 41

Formulas (117) may be derived from Fig. 41, by regarding P M m as a plane triangle, and Z m = Z M. The first produces no error greater than 0″.5. The error of the second is evidently greater the greater the altitude, or the latitude. This error, however, will not be more than 0′.5 in latitudes less than 20°, nor more than 2′ in latitudes less than 60°.

213. We may use (116) with more exactness, but these formulas may be modified so as to facilitate computation.

Put $$\phi'' = h + \Delta h$$

then, changing the 2d of (116) to a logarithmic form, we have

$$\log \sin (h + \Delta h) = \log \sin h + \log \cos \phi - \log \cos p,$$

or

$$\log \sin (h + \Delta h) - \log \sin h = \log \sec p - \log \sec \phi.$$

But Δh being very small, representing by $D_{''}$ the change of log sin h for 1″, we have, with Δh in *seconds*,

$$\log \sin (h + \Delta h) - \log \sin h = \Delta h \times D_{''};$$

whence, by substituting in the preceding, we obtain

$$\Delta h = \frac{\log \sec p - \log \sec \phi'}{D_{''}} = \frac{\log \cos \phi - \log \cos p.}{D_{''}} \qquad (118)$$

The difference of the log secants, or log cosines, of p and ϕ is readily taken from the table by inspection. $D_{''}$ for log sin h is usually given in tables of 7 decimal places, and hence Δh is readily found.

We have then
$$\left.\begin{aligned} \phi &= p \cos t \\ L &= h + \Delta h - \phi \end{aligned}\right\} \qquad (119)$$

If $D_{'}$ is the change of log sin h for 1′, then in *minutes*

$$\Delta h = \frac{\log \sec p - \log \sec \phi.}{D_{'}} \qquad (120)$$

Altitudes of Polaris may often be observed at sea, with some degree of precision, during twilight, when the horizon is well defined, and the latitude found from them within 3′ or 4′.

EXAMPLE 1: March 31, 1918, in Lon. 160° 15′ E., the sextant altitude of Polaris was 38° 18′ 00″, I. C. + 3′ 00″, height of eye 17 feet, W. T. of observation $7^h\ 15^m\ 19^s$ P.M., C − W $1^h\ 22^m\ 17^s$, chronometer fast $3^m\ 17^s$. Find the latitude.

Time				Alt. and Lat.				Alt. Corr.		R. A. M. S.				*'s R. A. and Dec.
	h	m	s		°	′	″	′	″		h	m	s	*'s R. A. $1^h\ 29^m\ 56^s.8$
W.	7	15	19	Obs.	38	18	00	I. C.	+3	G. M. N.	0	28	15.2	*'s Dec. 88° 52′.3 N.
C − W.	1	22	17	Corr.	−	2	16	Dip.	−4	Corr. G. M. T.	+	3	22.8	p = 1° 07′.7
C.	8	37	36	*'s h	38	15	44	r	−1		0	31	38	= 67′.7
$c.c.$		− 3	17	ϕ =		5	00	Corr.	−2					
G. M. T. Mar. 30	20	34	19	Lat. =	38	20	44 N.							
R. A. M. S. =	0	31	38											
G. S. T.	21	05	57											
Lon. E.	10	41	00											
L. S. T.	7	46	57											
*'s R. A.	1	29	57											
t =	6	17	00											
t =	94°	15′	00″											

p = 67.7	log	1.83059
t = 94° 15′	cos	−8.86987
ϕ = − 5′.017	= log	0.70046

214. ***Find the latitude by an observed altitude of Polaris,*** (N. A. Method).

Reduce the observed altitude of Polaris to the true altitude.

Reduce the recorded time of observation to the L. S. T.

With this sidereal time take out the correction from the table below, and add it to or subtract it from the true altitude, according to its sign. The result is the approximate latitude of the place.

EXAMPLE: June 10, 1918, at $10^h\ 40^m\ 30^s$ P.M., L. M. T., in Lon. 74° W., the true altitude of Polaris is 39° 46′. Find the latitude.

	h	m	s
Local astronomical mean time	10	40	30
Reduction (Tab. III), for $10^h\ 40^m\ 30^s$	+	1	45
R. A. M. S., June 10		5	12	7
Red. (Tab. III), for longitude (= $4^h\ 56^m$ West)	+	0	49
Sum (having regard to signs) is equal to L. S. T.	15	55	11
		°	′
True altitude		39	46
Correction from table below	+	0	55
Approximate latitude	+	40	41

EXTRACT FROM TABLE I. (N. A.)

Local S. T.	15^h	16^h	17^h
m	° ′	° ′	° ′
0	+1 2.7 11	+0 54.1 18	+0 41.9 23
10	1 1.6 13	0 52.3 19	0 39.6 25
20	1 0.3 14	0 50.4 20	0 37.1 25
30	+0 58.9 15	+0 48.4 20	+0 34.6 25
40	0 57.4 16	0 46.4 22	0 32.1 26
50	0 55.8 17	0 44.2 23	0 29.5 27
60	+0 54.1	+0 41.9	+0 26.8

215. ***To find the latitude by the rate of change of altitude near the prime vertical*** (Prestel's Method.)*

In the note to Art. 249 we have, for a very brief interval of time, and a small change of altitude,

$$\mathrm{d}\,t = \frac{\mathrm{d}\,h}{15 \cos L \sin Z},$$

or,

$$T' - T = t = \frac{h' - h}{15 \cos L \sin Z};$$

* Chauvenet's Astronomy, Vol. I.

whence $$\cos L = \frac{h' - h}{15\,t} \operatorname{cosec} Z; \qquad (121)$$

in which $h' - h$ is expressed in seconds of arc and t in seconds of time, and, Z being + when *east*, − when *west*, $\cos L$ is always positive. If Z is near 90°, its cosecant varies slowly. When $Z = 90°$ we have,

$$\cos L = \frac{h' - h}{15\,t}. \qquad (122)$$

If, then, two altitudes are carefully observed near the prime vertical, and the times noted with great precision, the interval not exceeding 8 or 10 minutes, an approximate latitude may be found by (122), when the altitudes are within 2° or 3° of the prime vertical; or by (121) when they are at a greater distance, and Z is approximately known.

The time of passing the prime vertical can be found by (91). Z may be roughly computed from the altitudes, or found within 2° from the bearing observed by a compass, which will suffice, if the observations are made within 10° of the prime vertical.

As, near the prime vertical, the altitude changes uniformly with the time, several altitudes may be observed in quick succession, and the mean taken as a single altitude.

The larger $h' - h$ and t, consistent with the supposition of uniformity of change and the condition by which they are substituted for their trigonometric functions, the more accurate in general will be the result.

Still the method does not admit of much precision. It is entirely unavailable near the equator, and in latitude 45° may give a result in error from 5 to 10 minutes, even when the greatest care has been bestowed on the observations. It may, however, be useful to the navigator in high latitudes, as it can be used for altitudes of the sun, when almost exactly east or west, and it will restrict the position of the ship to a limited portion of the line of position found by Sumner's Method. (Art. 256.) There are occasions at sea, when to find the latitude only within 10′ is very desirable.

EXAMPLE 1: June 15, 1918, in Lat. 60° N., Lon. 60° W., by D. R. observed with a sextant at W. T. $7^h\ 13^m\ 26^s$ A.M. an altitude of the sun's lower limb 27° 00′ 23″ and at W. T. $7^h\ 19^m\ 51^s$ A.M. observed another altitude of the sun's lower limb 27° 48′ 42″ bearing of sun by compass N. 88° E. Find the latitude.

Time				Altitudes				Azimuth
	h	*m*	*s*		°	′	″	N. 88° E.
W.	7	13	26	1st obs. *h*	27	00	23	
W.′	7	19	51	2d obs. *h*′	27	48	42	
W. − W.′		6	25	*h* − *h*′		48	19	
t		= 385^s				= 2899″		

15		
h − *h*′ = 2899″	colog	8.82391
t 385^s	log	3.46225
	colog	7.41454
Z 88°	cosec	0.00026
L = 59° 51′ N.	cos	9.70096

EXAMPLE 2: May 14, 1918, at sunset, in Lat. 43° 58′ N., Lon. 40° 10′ W., by D. R., the bearing of the sun was W. (true). The sun's lower limb was brought tangent to the horizon at $7^h\ 07^m\ 00^s$ P.M. watch time; the sextant was kept clamped until the upper limb was tangent to the horizon, W. T. $7^h\ 09^m\ 57^s$ P.M. Find the latitude.

	h	*m*	*s*
W.	7	07	00
W.′	7	09	57
W. − W.′		2	57
t		= 177^s	

		′	″
Semi-dia., May 1 (Eph.)		15	54.06
			2
Diameter		31	48.12
	= 1908″.12 = *h* − *h*′		

15		
t = 177^s	colog	8.82391
h − *h*′ = 1908.12	colog	7.75203
	log	3.28061
Lat. = 44° 03′ N	cos	9.85655

CHAPTER XII

THE CHRONOMETER — LONGITUDE

216. ASTRONOMICALLY the longitude of a place is the difference of the local and Greenwich times of the same instant. It is *west* or *east*, according as the Greenwich time is greater or less than the local time. (Art. 115.)

The *mean solar*, the *apparent*, or the *sidereal* times of the two places may be thus compared.

217. A *chronometer* is simply a correct time-measurer, but the name is technically applied to instruments adapted to use on board ship. It is here used more generally, as including clocks which are compensated for changes of temperature.

A *mean time* chronometer is one regulated to mean time; that is, so as to gain or lose daily but a few seconds on mean time.

A *sidereal* chronometer is one regulated to sidereal time.

218. A chronometer is said to be regulated to the local time of any place when it is known how much it is too fast, or too slow, of that local time, and how much it gains or loses *daily*. The first is the *error* (on local time); the second is the *daily rate*. Both are + if the chronometer is *fast* and *gaining*.

It is preferable, however, to use the *correction* of the chronometer, which is the quantity to be applied to the chronometer time to reduce it to the true time, and its *daily change*. Both are + when the chronometer is *slow* and *losing*.

They will be designated by c and Δc.

A chronometer is said to be regulated to Greenwich time when its corrections on Greenwich time and its daily change are known.

If c_0 is the chro. cor. to reduce to Greenwich time, and c the chro. cor. to reduce to the time of a place whose longitude is λ (+ if west).

$$c_0 = c + \lambda, \quad \text{or} \quad c = c_0 - \lambda; \tag{123}$$

so that the one can readily be converted into the other.

219. If the correction of the chronometer at a given date,

and its daily change, are known, the correction at another date can easily be found. For let

c be the given correction at the date T,
c', the required correction at the date T'
$t = T' - T$, expressed in days,
Δc, the daily change;

then
$$c' = c + t\,\Delta c. \tag{124}$$

t is negative if the date for which the correction is required is before that for which it is given.

If Δc is large, t must include the parts of a day in the elapsed time.

Δc may be given for two different dates, and vary in value. It may then be interpolated for the middle date between the two of this problem.

Thus, if $\Delta' c$ be a second value determined n days after the first, the daily variation of Δc, regarded as uniform, will be

$$\frac{\Delta c - \Delta c}{n}. \tag{125}$$

Representing this by $\Delta_2 c$, we have for the mean daily change of the chronometer correction during the period t, or that at the middle date,

$$\Delta c + \tfrac{1}{2}\, t\, \Delta_2 c,$$

and the required chronometer correction,

$$c' = c + t\,\Delta c + \tfrac{1}{2}\, t^2\, \Delta_2 c. \tag{126}$$

When the chronometer is in daily use, it is convenient to form a table of its correction for each day at a particular hour. For a stationary chronometer, the most convenient hour is 0^h of local time; for a Greenwich chronometer, 0^h of Greenwich time.

EXAMPLE 1 : Chro. 1675, regulated to Greenwich mean time; January 15, 1918, 0^h; correction $+\ 1^h\ 16^m\ 25^s.0$; daily change $-\ 7^s.65$. Find the correction, January 26, 6^h.

Chro. corr., Jan. 15 , 0^h	$+\ 1^h\ 16^m\ 25^s.0$
Change $11\frac{1}{4}d.$ $= -7^s.65 \times 11.25 =$	$-\ \ 1\ \ 26\ .1$
Chro. corr., Jan. 26, 6^h	$+\ 1\ \ 14\ \ 58.9$

This chronometer is *slow* and *gaining*.

EXAMPLE 2: To find the chronometer correction to reduce to local time, January 26, 1918, 0^h, in Lon. 85° 16′ E.

Chro. corr. (Jan. 26 6^h G. t.)		$+1^h\ 14^m\ 58^s.9$	Rate $-7^s.65$
− Lon.	+6	+5 41 4	$-12^h = 3^s.8$
Red. for	−12	+3.8	
Chro. corr. (Jan. 26 0^h L. t.)		+6 56 6.7	or $-5^h\ 3^m\ 53^s.3$

EXAMPLE 3: To form a table of chronometer correction for each day from January 26, 1918, 6^h, to February 6, 6^h.

G. M. T.	Chro. corr.	G. M. T.	Chro. corr.
Jan. 26 6^h	$+1^h\ 14^m\ 58^s.9$	Feb. 1 6^h	$+1^h\ 14^m\ 13^s.0$
27 6	14 51 .3	2 6	14 5 .4
28 6	14 43 .6	3 6	13 57 .7
29 6	14 36 .0	4 6	13 50 .1
30 6	14 28 .3	5 6	13 42 .4
31 6	+1 14 20 .7	6 6	+1 13 34 .8

220. To find the *rate*, or *daily change*, of a chronometer, it is necessary to find the correction of the chronometer on two different days, either from observations, or by comparison with a chronometer whose correction is known. Let c_1 and c_2 be the two corrections, t the interval expressed in days; then we have for the daily change,

$$\Delta c = \frac{c_2 - c_1}{t} \qquad (127)$$

that is, the daily change is equal to the difference of the two chronometer corrections divided by the number of days and parts in the interval. If attention is paid to the sign, + will indicate that the chronometer is *losing*, − that it is *gaining*.

EXAMPLE:

	Chro. 1615			Chro. 4872			Chro. 796		
	h	m	s	h	m	s	h	m	s
Chro. corr. April 15 0	+0	18	16.2	−1	15	27.5	+0	0	16.6
Chro. corr. April 27 8	+0	18	29.6	−1	14	58.6	−0	0	5.3
Change in 12.3 days			+13.4			+28.9			−21.9
Daily change of corr.			+1.09			+2.35			−1.71

At fixed observatories an interval of one day may suffice. For rating sea-chronometers by observations made with a sextant and artificial horizon, an interval of from 5 to 15 days is desirable.

The sea-rate of a chronometer is sometimes different from its rate on shore, or even from its rate while on board a vessel in port. Some chronometers are affected by magnetic influences; all are slightly affected by changes of temperature, as perfect compensation is rarely attainable. The excellence of

a chronometer depends upon the permanence of its rate. The rate may be large, but if its variations are small the chronometer is good.

221. A watch is often used for noting the time of an observation. It is compared with the chronometer by noting the time of each at the same instant. The most favorable instant is when the watch shows 0^s.

Let C and W be these noted times; then $\Delta W = (C - W)$ is the reduction of the watch time to the chronometer time for $C = W + (C - W)$.

It is considered good practice always to subtract watch time from chronometer time, whatever the relative values, and thus to employ $C - W$ invariably as an additive correction. It is equally correct to take the other difference, $W - C$, and make it subtractive; it may sometimes occur that a few figures will thus be saved, but a change for error arises from the possibility of inadvertently using the wrong sign, which is almost impossible by the other method.

System of daily comparisons. — In order to determine whether the chronometers are working steadily, recourse is had to a system of daily comparisons the results of which are entered in a book called the chronometer journal. This system consists of comparisons between the A chronometer and all the other chronometers and deck watches in the vessel, and is carried out as follows:

(Chronometers beat every half second.)

When about to compare, only open the lid of A chronometer, so that its tick may sound loudly and that of the others may be deadened. Write down a time which A is going to show, say $1^h\ 13^m\ 40^s$, and start counting the beats when the second hand gets to 30 seconds, thus: Half, one, half, two, half, three, etc.; in a few moments the beats can easily be counted; then still counting the beats turn the eye to the other chronometer, and note exactly what its second hand shows at the instant you hear A beat the exact second decided on. Write down the comparison as follows:

	h	*m*	*s*
Stand. *A*,	1	13	40
Chro. *B*,	1	12	21.5
Diff.		1	18.5 slow on *A*.

Comparisons should be made before and after the observation, and the results interpolated to the time of observation.

A practised observer may, by looking at the watch and counting the beats of the chronometer, make the comparison to the nearest $0^s.25$. It is better to take the mean of several comparisons than to trust to a single one.

A mean time and a sidereal chronometer may be compared within $0^s.03$ by watching for the coincidence of beats which occurs at intervals of 3^m, for chronometers, which beat half-seconds.

222. The following form represents a good method of recording comparisons:

Stand. A, No. 5		Chro. B, No. 150			Chro. C, No. 802							
Date, 1918	Designation of comparisons	Chro. B with stand. A	2d diff.	Chro. C with stand. A.	2d diff.	Therm. Max.	Therm. Min.	Therm. Air.	Bar.	Remarks
		h m s	*s*	*h m s*	*s*	°	°	°	"	
January 1	Stand. A	1 13 40		1 14 20		63	59	60	30.07	Found errors by timeball.
	B and C	1 12 21.5		2 04 11						Strong breeze. Ground swell.
	Diff.	1 18.5	−	11 10 09	−					
2	Stand. A	1 16 30		1 17 00		64	58	57	30.12	Left San Francisco for Honolulu.
	B and C	1 15 10		2 06 51.5						
	Diff.	1 20	+1.5	11 10 08.5	−0.5					

The daily difference between the comparisons of any two chronometers is the difference between their daily rates; any alteration of the daily difference shows that one or both of the chronometers is running irregularly. If the daily difference of comparison between *A* and *B* remains constant at the approximate algebraic sum of their previously obtained daily rates,

and that between A and C alters, it is probable that the rate of C chronometer has altered; thus, when finding the error of the deck watch from the chronometer journal in order to determine a position line, it is necessary to examine as to whether all the chronometer agrees, and this is done as follows.

Apply to the estimated error of B, found as explained above, the comparison between A and B, and thus find the error of A on G. M. T., as indicated by the B chronometer. Similarly the error of A on G. M. T. as indicated by other chronometers may be found. Thus we find three or more possible errors for the A chronometer, an inspection of which will show the most probable error of A, and from this the error of the watch is determined. In the event of no one error appearing more probable than another, their mean should be assumed to be the error of A. If the predicted rates of the chronometers were such as to give for the second difference of $A - B$, $+ 1^s.5$, and of $A - C$, $- 0^s.5$, suppose on a certain day those differences were $+ 4^s.5$ and $- 0^s.5$, respectively; it would at once be suspected that the irregularity was in B, and that that chronometer had lost 3^s on its normal rate during the preceding day. Suppose, however, the second differences were $+ 4^s.5$ and $+ 2^s.5$; it would then be apparent that A had gained 3^s.

If a landfall has to be made, or a danger to be cleared, from a position obtained by astronomical observations, when there is a considerable disagreement between the several chronometers as to the error of A, that error should be selected which places the vessel in the most disadvantageous position.

EXAMPLE:

	Chro. 476			Chro. 4072			Chro. 4072			Chro. 1976.		
	h	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*
Chro.	4	16	56.2	3	15	17.5	3	15	17.5	1	0	28.5
Watch	1	5	0	7	35	30	7	35	30.0	4	28	0
$C - W.$	+3	11	56.2	−4	20	12.5	+7	39	47.5	+8	32	28.5

223. *The mean comparison.* — It is useless to obtain the G. M. T., however accurately done, unless it can be carried accurately to the chronometers. Suppose A has a steady gaining rate of 7^s a day and the watch a losing rate of 10^s a day, it is obvious that the comparison between them cannot remain constant for as much as an hour. When the rates are steady, it follows that if comparisons are made before landing, when necessary to do so to obtain the error, and again after returning, a comparison may be deduced which would be correct for any particular instant between the two comparisons actually

taken; this computed comparison is called a mean, and is found as follows:

Comparison before landing:—	
Time by A	$9^h\ 55^m\ 00^s$
Time by Watch	9 40 07 .5
W. slow on A	14 52 .5
Comparison after return:—	
Time by A	$11^h\ 56^m\ 00^s$
Time by W.	11 41 09 .5
W. slow on A	0 14 50 .5

Elapsed time between comparisons by W. = $2^h\ 01^m\ 02^s = 121^m$.

W. time at which error was observed	$11^h\ 13^m\ 19^s$
W. time of last comparison	11 41 09
Elapsed time	0 27 50 = $27^m.8$

By first comparison W. is $14^m\ 52^s.5$ slow on A,
By second comparison W. is 14 50 .5 slow on A;
therefore in 121^m the W. has gained 2^s on A and in $27^m.8$ the W. will gain

$$\frac{27.8 \times 2}{121} = .459^s.$$

By the second comparison the W. was $14^m\ 50^s.5$ slow on A; therefore, at the time at which the error was observed it was $0^h\ 14^m\ 50^s.959$ slow on A, which is the mean comparison.

224. ***To find the correction of a chronometer at a place whose latitude and longitude are given.***

First Method. (By *single* altitudes.)

Observe an altitude, or set of altitudes, of the sun or a star, noting the time by the chronometer, or a watch compared with it.

Find from the altitude (Art. 180) the local mean, or sidereal, time, as may be required.

The "local time" — the "chronometer time," or

$$c = T - C$$

(Art. 183), is the correction of the chronometer on local time. Applying to this the known longitude of the place of observation, gives the correction on Greenwich time.

225. If an artificial horizon is used, as it should be when practicable, it is best to make two sets of observations with the roof in reversed positions. In A.M. observations of the sun with a sextant and artificial horizon, the lower limb of the sun and the upper limb of its image in the horizon are made to lap, and the instant of separation is watched for; while in P.M. observations the limbs are separated and ap-

proaching, and the instant of contact is noted. In observations of the upper limb this is reversed. Even a good observer may estimate the contact of two disks differently when they are separating and when they are approaching. Both limbs, then, should be observed.

In observing altitudes which change rapidly it is better, when circumstances permit, to set the instrument so as to read exact divisions at regular intervals, and watch the instant of contact. A good observer, with a sextant and artificial horizon, can observe the double altitudes at regular intervals of 10′.

226. On a subsequent day repeat this observation, and find again the correction of the chronometer. The difference between these two corrections divided by the number of days and parts in the interval is the *daily change*, as in Art. 220.

It is important that both the observations thus compared should be at nearly the same altitude and on the same side of the meridian (when the sun is observed, both in the forenoon, or both in the afternoon), and in general, that they should be made with the same instruments, and as nearly as practicable under the same circumstances. Thus, an error in the assumed latitude and *constant* errors of the instruments or the observer will effect the two chronometer corrections nearly alike, but will very slightly affect their difference, and, consequently, the rate determined from it will be nearly exact. The chronometer correction, derived from single altitudes, may be erroneous a few seconds. But for sea chronometers this is of less importance than an erroneous determination of the rate. For instance, suppose the determined chronometer correction in error 4ˢ, and the daily change in error 1ˢ; in 20 days (Art. 219) the computed change of the correction will be in error 20ˢ, and in 30 days will be in error 30ˢ.

227. *Second Method.* (By *double* altitudes.)

It is better to observe altitudes of the body on both sides of the meridian, and as nearly at the same altitude as practicable, either on the same day or on two consecutive days.

Altitudes of two stars also may be used, one east, the other west of the meridian.

The mean of the two results is better than a determination from either alone; for constant errors of the latitude, the instrument, or the observer, affect the two results in opposite directions; that is, if one result is too large, the other is too small, and by nearly the same amount.

Chronometer Correction. EXAMPLE: May 30, 1918, at Pensacola, Fla., Lat. 30° 20′ 30″ N. Lon. 87° 15′ 21″ W., observed an altitude of the sun for chronometer error using a sextant and artificial horizon, sun's limbs closing, mean of double sextant's altitudes of sun's upper limb was 100° 15′ 00″, I. C. + 3′ 00″, W. T. of observation $9^h\ 02^m\ 03^s$ A.M., chronometer fast $5^m\ 07^s$ (approx.). C.− W, $5^h\ 53^m\ 15^s$. Find chronometer correction. (Use Ephemeris.) (See page 516 Eph.)

Time

	h	m	s
W.	9	02	03
C − W	5	53	15
C	2	55	18
c. c.		−5	07
G. M. T.	2	50	11
May 31	= $2^h.84$		

Altitudes

		°	′	″
2	⊙	100	15	00
I. C.		+	3	00
2		100	18	00
		50	09	00
Corr.		−	16	31
		49	52	29

Corr. to Alt.

		″
S. D.	−15	48
p & r	−	43
Corr.	−16	31

Dec.

	°	′	″
	+21	50	25.4
Corr.	+	1	02.14
N.	21	51	27.54
p =	68	08	32

H. D. +21″.88
G. M. T. $2^h.84$
Corr. +62″.139

Eq. of Time

	h	m	s
Lon. place	+5	49	01.4
Lon. Washington	+5	08	15.8
	+	40	45.6 W. of Washington

	h	m	s
L. A. T.	21	04	09
λ W. of Wash.	+	40	46
W. A. T., May 30	21	44	55
or $-2^h.25$, May 31			

	m	s
Eq. t. W. A. N., May 31	−2	36.47
Corr.	+	.79
(− to app. t.)	−2	37.26

H. D. −0.351
W. A. T. = $-2^h.25$
Corr. +.78975

	°	′	″
h	49	52	29
L	30	20	30
p	68	08	32
2	148	21	31
s	74	10	46
h	49	52	29
$s - h$	24	18	17

sec 0.06398
cosec 0.03240
cos 9.43556
sin 9.61446
2 | 91464.0
sin 9.57320

	h	m	s
L. A. T.	21	04	09.55
Eq. t.	−	2	37.26
L. M. T.	21	01	32.29
Lon. W.	5	49	01.4
G. M. T.	2	50	33.69
Chro.	2	55	18.00
Chro. fast		4	44.31

228. *Third Method.* (By *equal* altitudes.)

A heavenly body which does not change its declination is at the same altitude east and west of the meridian at the same interval of time from its meridian passage.

If, then, such equal altitudes are observed and the times noted by the chronometer, or by a watch and reduced to the chronometer (Art. 221), the mean of these times, or the *middle time*, is the chronometer time of the star's meridian transit.

The corresponding sidereal time is the star's right ascension, when the first observation is east of the meridian; 12^h + the right ascension when the first observation is west of the meridian.

This, for a mean time chronometer, may be converted into local mean time (Art. 143); and for a Greenwich chronometer into the corresponding Greenwich time.

Subtracting the chronometer time, we have the correction of the chronometer.

EXAMPLE: January 14, 1918, at Washington, in longitude $5^h\ 08^m\ 15^s.78$, equal altitudes of Procyon (α Canis Min.) were observed, and the times noted by a chronometer regulated to Greenwich mean time; from which were obtained:

	h	*m*	*s*
Mean of Chro. times (* east)	2	17	35.65
Mean of Chro. times (* west)	8	00	16.38
Chro. time of *'s transit	5	08	56.01
L. sid. t. = *'s R. A.	7	35	03.18
Lon. W.	+5	08	15.78
G. sid. t.	12	43	18.96
R. A. M. S., Jan. 14.	19	32	33.67
Sid. int. from Jan. 14 0^h	17	10	45.29
Red. (Tab. II)		2	48.86
G. M. T., Jan. 14.	17	07	56.43
Chro. time	17	08	56.01
Chro. fast on G. M. T.		0	59.58

229. If equal altitudes of the sun are observed in the forenoon and afternoon of the same day, the mean of the noted times would be the chronometer time of *apparent noon*, were it not for the change of the sun's declination between the observations.

In equal altitudes of the sun, to find the correction of the middle time for the change of the sun's declination in the interval between the observations.

Solution. Let

h = the sun's true altitude at each observation,
t = half the elapsed *apparent* time between the observations,
T_0 = the mean of the chronometer times of the two observations, or the *middle* chronometer time,
ΔT_0 = the correction of this mean to reduce to the chronometer time of apparent noon;
L = the latitude of the place,
d = the sun's declination at local apparent noon,
Δd = the change of this declination in the time t;

then, when both observations are on the same day,

$t + \Delta T_0$ will be numerically the hour-angle at the A.M. observation,
$t - \Delta T_0$, the hour-angle at the P.M. observation,
$d - \Delta d$, the declination * at the A.M. observation,
$d + \Delta d$, the declination * at the P.M. observation.

By (71), we have for the two observations,

$$\left.\begin{aligned} \sin h &= \sin L \sin (d-\Delta d) + \cos L \cos (d-\Delta d) \cos (t+\Delta T_0) \\ \sin h &= \sin L \sin (d+\Delta d) + \cos L \cos (d+\Delta d) \cos (t-\Delta T_0) \end{aligned}\right\} \quad (128)$$

But

$$\begin{aligned} \sin (d \pm \Delta d) &= \sin d \cos \Delta d \pm \cos d \sin \Delta d, \\ \cos (d \pm \Delta d) &= \cos d \cos \Delta d \mp \sin d \sin \Delta d, \\ \cos (t \pm \Delta T_0) &= \cos t \cos \Delta T_0 \mp \sin t \sin \Delta T_0. \end{aligned}$$

Since Δd, and therefore ΔT_0, are very small, we may put

$$\begin{aligned} \cos \Delta d &= 1, & \sin \Delta d &= \Delta d \sin 1'', \\ \cos \Delta T_0 &= 1, & \sin \Delta T_0 &= 15\, \Delta T_0 \sin 1''; \end{aligned}$$

Δd being expressed in seconds of arc, and ΔT_0 in seconds of time; we shall then have

$$\begin{aligned} \sin (d \pm \Delta d) &= \sin d \pm \Delta d \sin 1'' \cos d, \\ \cos (d \pm \Delta d) &= \cos d \mp \Delta d \sin 1'' \sin d, \\ \cos (t \pm \Delta T_0) &= \cos t \mp 15\, \Delta T_0 \sin 1'' \sin t. \end{aligned}$$

Substituting these in the two equations (128), subtracting the first from the second, and dividing by $2 \sin 1''$, we shall have

* Strictly, in the one case, Δd should be the change of declination in the time $t + \Delta T_0$; in the other, the change in the time $t - \Delta T_0$.

$$0 = \Delta d \sin L \cos d - \Delta d \cos L \sin d \cos t + 15\,\Delta T_0 \cos L \cos d \sin t.$$

Transposing and dividing by the coefficient of ΔT_0, we find the formula

$$\Delta T_0 = -\frac{\Delta d \tan L}{15 \sin t} + \frac{\Delta d \tan d}{15 \tan t}, \qquad (129)$$

which is called the equation of equal altitudes.

Let

$\Delta_h d$ = the hourly change of declination at the instant of apparent noon, and express

t, which is half the elapsed apparent time, in hours,

then $$\Delta d = \Delta_h d\, t,$$

and (129) becomes

$$\Delta T_0 = -\frac{\Delta_h d\, t \tan L}{15 \sin t} + \frac{\Delta_h d\, t \tan d}{15 \tan t}. \qquad (130)$$

If we put

$$A = -\frac{t}{15 \sin t}, \qquad B = \frac{t}{15 \tan t}. \qquad (131)$$

and

C_0 = the chronometer time of apparent noon, we have

$$\left.\begin{aligned} \Delta T_0 &= A\,\Delta_h d \tan L + B\,\Delta_h d \tan d \\ C_0 &= T_0 + \Delta T_0 \end{aligned}\right\} \qquad (132)$$

In these formulas, L and d are + when *north*, Δd and $\Delta_h d$ are + when the sun is moving toward the *north*.

The coefficient A is −, since $t < 12^h$,
" " B is + when $t < 6^h$, − when $t > 6^h$.

The computation of the two parts of ΔT_0 is facilitated by tables of log A and log B. Such tables are given in Chauvenet's "Method of Finding the Error and Rate of a Chronometer."

The argument of these tables is $2t$, or the elasped time. The signs of A and B are given.

Apply the two parts of ΔT_0, according to their signs, to the *Middle Chronometer Time;* the result is the *Chronometer Time of Apparent Noon.*

Apply to this the equation of time (*adding*, when the equation of time is *additive*, to *mean* time; otherwise *subtracting*); the result is the *Chronometer Time of Mean Noon* at the place.

Applying to this the longitude (in time), *subtracting* if *west*, *adding* if *east*, gives the *Chronometer Time of Mean Noon at Greenwich.*

* 12^h – *Chro. T. at local Mean Noon* will be the *Chro. correction*, if the chronometer is regulated to local time.

12^h – *Chro. T. at Greenwich Mean Noon* will be the *Chro. correction*, if the chronometer is regulated to Greenwich time.

230. If a set of altitudes is observed in the afternoon of one day, and a set of equal altitudes in the forenoon of the next day, the middle time would correspond nearly to the instant of *apparent midnight;* and half the elapsed time t would be nearly the hour-angle from the lower branch of the meridian, or the supplement of the proper hour-angle.

In this case

$180° - (t + \Delta T_0)$ will be the hour-angle at the P.M. observation.
$180° - (t - \Delta T_0)$ " " " " " " A.M. "
$d - \Delta d$, the declination at the P.M. "
$d + \Delta d$, " " " " A.M. "

and we have for the two observations, as in (128)

$$\left.\begin{aligned} \sin h &= \sin L \sin (d-\Delta d) - \cos L \cos (d-\Delta d) \cos (t+\Delta T_0) \\ \sin h &= \sin L \sin (d+\Delta d) - \cos L \cos (d+\Delta d) \cos (t-\Delta T_0) \end{aligned}\right\} \dagger (133)$$

Treating these in the same way as (128) we shall have

$$0 = \Delta d \sin L \cos d + \Delta d \cos L \sin d \cos t - 15\, \Delta T_0 \cos L \cos d \sin t;$$

whence

$$\Delta T_0 = \frac{\Delta d \tan L}{15 \sin t} + \frac{\Delta d \tan d}{15 \tan t}$$

or, putting as before $\Delta d = \Delta_h d\, t$

* This is better noted as 0^h.

† These may be written

$$-\sin h = -\sin L \sin (d - \Delta d) + \cos L \cos (d - \Delta d) \cos (t + \Delta T_0)$$
$$-\sin h = -\sin L \sin (d + \Delta d) + \cos L \cos (d + \Delta d) \cos (t - \Delta T_0).$$

They differ from (128) in the signs of h and L, and in reckoning the hour-angles from the lower, instead of the upper, branch of the meridian. This would be the case, if we suppose the observations to be referred to the latitude and meridian of the antipode. The only effect in (129) is to change the sign of tan L, or of the first term in the equation of equal altitudes.

$$A = -\frac{t}{15 \sin d}, \qquad B = \frac{t}{15 \tan t},$$

$$\Delta T_0 = -A \Delta_h d \tan L + B \Delta_h d \tan d, \qquad (134)$$

which differs from (132) only in the sign of A. This is the reduction of the middle time to the *Chro. Time of apparent midnight;* applying the equation of time reduces it to the *Chro. Time of mean midnight.*

231. d, Δd, and the equation of time, are to be taken from pages 514 to 521 for Washington apparent noon of the Ephemeris, and interpolated for the instant of apparent noon, or of apparent midnight, according as the observations are made on the same day, or on consecutive days.

$2t$ is properly the *elapsed apparent time.* The elapsed time by chronometer requires, then, not only a correction for the rate, which is

$$\frac{2t}{24^h} \Delta c \ (+ \text{ when the chronometer loses});$$

but also a reduction to an apparent time interval, which, for a mean time chronometer, is the change * of the equation of time in the time, $2t$, additive when the equation of time is additive to *mean* time and increasing, or subtractive from mean time and decreasing. For a sidereal chronometer, it is the change in the sun's right ascension in the time $2t$, and subtractive.

232. Equal altitudes of the moon or a planet may be observed; but in the case of the moon admit of less precision than of the sun, and moreover require correction for the inequality produced by change of parallax.

If $2 \Delta a$ is the *increase* of right ascension in the interval, the body will arrive at its second position later than would a fixed star, supposed coincident with it at the first position; and the elapsed *sidereal* time will be greater than the double hour-angle of the body by the quantity $2 \Delta a$. If $2s$ = the elapsed *sidereal* time, then in (130) we must take

$$2t = 2s - 2 \Delta a, \text{ or } t = s - \Delta a. \qquad (135)$$

If t_m = half the elapsed *mean* time (expressed in hours when used as a coefficient), and

$\Delta_h a$ = the *increase* of right ascension in 1^h of *mean* time,

* The maximum daily change is 30ˢ. The elapsed time by Chronometer is usually regarded as sufficiently accurate.

by (59) $s = t_m + 9^s.8565\ t_m$
and $t = t_m + t_m\ (9^s.8565 - \Delta_h\ a),$ (136)

by which t and $2\ t$ may be found from $2\ t_m$ the elapsed mean time.

In this expression the last two terms are in seconds. Reducing to hours we have

$$t = t_m \left(1 + \frac{9^s.8565 - \Delta_h\ a}{3600}\right) = t_m \left(1.002738 - \frac{\Delta_h\ a}{3600}\right) \quad (137)$$

If $\Delta_h\ d$ = the change of declination in 1^h of mean time, then in (129)

$$\Delta\ d = t_m\ \Delta_h\ d$$

or, substituting for t_m its value from (137),

$$\Delta\ d = t\ \Delta_h\ d \div \left(1.002738 - \frac{\Delta_h\ a}{3600}\right).$$

Equations (131) and (132) may then be used for other bodies than the sun, provided we give t its proper value from (135) or (136), and for $\Delta_h\ d$ substitute

$$\Delta'_h\ d = \Delta_h\ d \div \left(1.002738 - \frac{\Delta_h\ a}{3600}\right),$$

or, which will be sufficiently exact,

$$\Delta'_h\ d = \Delta_h\ d + \frac{\Delta_h\ a - 9^s.856}{3600} \Delta_h\ d. \quad (138)$$

233. Observing the double altitudes at regular intervals of $10'$, or $20'$, especially facilitates the method of equal altitudes; for, if the first set is observed at equal intervals, in the second the observer, having set the instrument for the *last* reading of the first and observed the contact, for the subsequent observations has only to move back successively the same intervals.

234. It is not requisite that the instrument should give the true altitude; it is sufficient if the altitude is the *same* at the two corresponding observations. Hence the two observations should be made with the same instruments, without change of adjustment, and in general as nearly as practicable under the same circumstances.

This purpose is promoted by making the final movement of the tangent screw in both sets always in the same direction.

Thus, in reversing the movement, the screw may be turned a little too far, and then the final contact made by a motion in the same direction as before.

If the sun is used, both limbs should be observed.

The error arising from want of parallelism of the surfaces of the roof-glasses of the horizon is eliminated by having the *same* end of the roof toward the observer. The roof may be tested by observing sets of altitudes with it in reversed positions.

235. Although the readings of the instrument may be the same in the two sets of observations, the altitudes may be slightly different, 1st, from changes in the instrument in the interval; 2d, from difference of refraction at the two times.

A change in the index correction may be detected by observation; but there may be expansion or contraction of various parts of the instrument which may affect the readings of the altitudes without altering the index correction.

The change of refraction may be found by noting the barometer and thermometer at each set, and finding the refraction for both sets of altitudes.

236. To correct the middle time for any small difference of the altitudes, whether from refraction or actual change of readings, we may find, from the difference between two readings, and the difference of the corresponding times, the change of time for a change of $1'$, or $1''$, of altitude. This multiplied by half the inequality of altitudes, expressed in minutes, or seconds, will give the correction of the middle time, to be added when the P.M. altitude is the greater; to be subtracted when the P.M. altitude is the less.

If twice the altitude is observed with an artificial horizon, we may find the change of time for a change of $1'$, or $1''$, of the double altitude, and multiply it by the whole inequality of the altitudes.

EXAMPLE 1: January 10, 1918, at Key West, Fla., Lat. $24° 33' 20''$ N., Lon. $81° 48' 37''$ W., observed with a sextant, equal altitudes of the sun's lower limb, using an artificial horizon; mean of A.M. chronometer times $3^h\ 42^m\ 34^s.21$, mean of P.M. chronometer times $8^h\ 52^m\ 51^s.46$ I. C. $0'\ 00''$. Find the chronometer correction. (Use Ephemeris.)

λ, Key West = $5^h\ 27^m\ 14^s.5$ W.
λ_1 Washington = $5^h\ 08\ \ 15\ .8$ W.
$\lambda - \lambda$ = + $18^m\ 58^s.7$ W. of Washington = $0^h.316$ W.

Chro. Error and Time

	h	m	s
P. M. C. T.	8	52	51.46
A. M. C. T.	3	42	34.21
Mid. C. T.	6	17	42.835
1st and 2d parts		−	3.146
C. T. A. N.	6	17	39.689
Eq. of t.		−7	32.683
C.T. mean noon	6	10	07.006
Lon. W.	5	27	14.5
Chro. fast	0	42	52.506

t

	h	m	s
A. M. C. T.	3	42	34.21
P. M. C. T.	8	52	51.46
$2t$ =	18	49	42.75
t =	9	24	51.38
t =	$9^h.4143$		
t =	$141^\circ\ 12'\ 50''.7$		

	°	′	″			
L	24	33	20	tan	9.65982	
d	22	00	14.5			tan −9.60650
Δhd	22″.124			log	1.34487	log 1.34487
t	$9^h.4143$			log	0.97379	log 0.97379
t	$141^\circ\ 12'\ 51''$			cosec	0.20314	cot −0.09495
	15			colog	8.82391	colog 8.82391
	−1st part			log. +	1.00553	
				2d part +	6.9826	log +0.84402
				1st part −	10.1281	
					− 3.1455	

Declination

	h	′	″
Jan. 10 W. A. N. −	22	00	21.5
Corr. $\lambda - \lambda_1$		+	6.99
L. A. N.	S. 22	00	14.51

	″
H. D. dec. Jan. 10	+22.11
H. D. dec. Jan. 11	+23.18
Change 24^h	+ 1.07
Change 1^h	+ 0.044
Change $\lambda - \lambda_1$	+ 0.014
H. D. Jan. 10, W. A. N.	+22.11
Δhd,	+22.124
Change $\frac{\lambda - \lambda_1}{2}$ =	+ .007
H. D. Jan. 10	+22.11
H. D. (mean) dec.	+22.117

H. D. (mean)

	+22″.117
$\lambda - \lambda_1$	$0^h.316$
Corr.	+6′.9889

Eq. of T.

	m	s
Jan. 10, W. A. N.	− 7	32.36
Corr.	−	.323
(− to mean)	7	32.683

	s
H. D. Eq. t., Jan. 10	1.022
H. D. Eq. t., Jan. 11	0.998
Change 24^h	0.024
Change 1^h	.001
Change $\frac{\lambda - \lambda_1}{2}$	.000
H. D., Jan. 10, W. A. N.	1.022
H. D. Eq. t. (mean)	1.022

H. D. (mean)

	s
	−1.022
	$0^h.316$
Corr.	−.3229

237. *Fourth Method* of finding the correction of a chronometer. (By *transits.*)

On shore the most accurate method of finding the correction of a chronometer is by noting the times of transit of the sun or a star across the threads of a well-adjusted transit instrument. The mean of these times is taken and corrected for the errors of the instrument, or reduced to the meridian. In the case of the sun, the transits of both limbs may be observed.

At the instant of a star's transit of the meridian, the right ascension of the star is the *sidereal* time. The instant of transit of the sun's center is *apparent* noon.

From either of these, the local *sidereal* or *mean* time, as may be required, can be found; and thence the chronometer correction by subtracting the chronometer time of transit.

The moon should not be used for finding the time, when precision is required. Stars are preferred to the sun, either when transits are observed, or equal altitudes with the artificial horizon; chiefly because many stars may be observed during the same night, and the instrument is not exposed to the rays of the sun.

238. By repeating the transits on a subsequent day, the chronometer correction can be again found, and from the two corrections, the rate, as in Art. 220. If the transit instrument is not well adjusted, or the instrumental corrections are imperfectly known, the *rate* of the chronometer can still be quite well determined from transits of the same star, or the same set of stars on different days, provided the position of the instrument, or its adjustments, have not been disturbed in the interval.

239. *Fifth Method.* (By *time signals.*) These signals can be obtained at the W. U. Telegraph Office in any part of the United States, without delay, in any weather, and with absolute certainty as to comparisons. Since well-adjusted transit instruments are not generally available, the electric signal from Washington furnishes the best means of rating chronometers, with the utmost simplicity of method, and a high degree of accuracy. Time signals are also sent out by the high power radio stations to vessels at sea. A list of these stations and the time the signal is sent is given in the various Light Lists and Sailing Directions of the world.

At most important ports, for the convenience of vessels, a time ball, electrically connected with a clock, is dropped auto-

matically, at a certain G. M. T. When observing a time ball, about 10 seconds before the ball drops, count the seconds as follows: half, one, half, two, etc. Write down the time shown by the watch the instant the ball begins to fall and then immediately compare with the standard chronometer.

LONGITUDE

240. To find the longitude of a place by astronomical observations, it is generally necessary to determine independently the local and Greenwich times of the same instant. The difference of these times is the longitude, which is *west* when the Greenwich time is the greater, and *east* when the Greenwich time is the less (216). This is expressed by (53)

$$\lambda = T_0 - T,$$

in which T_0 is the Greenwich time, and

T, the corresponding local time of the same kind.

These times may be *apparent*, *mean*, or *sidereal*.

The apparent time is the hour-angle of the true sun; the mean time, that of the mean sun; the sidereal time, that of the vernal equinox. In the same way we may use the local and Greenwich hour-angles of any other body or point of the heavens regarded as + toward the *west*.

This is evident from Fig. 42; for if
P M is the meridian of Greenwich,
P M′, the local meridian,
P S, the declination circle of a heavenly body;
M P M′ will be the longitude of the place,

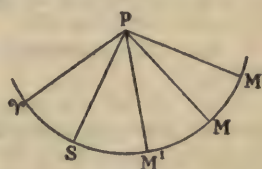

FIGURE 42

M P S, the hour-angle of the body at Greenwich,
M′ P S, the local hour-angle;

and we shall have, as in Art. 116,

$$\text{M P M}' = \text{M P S} - \text{M}' \text{P S}.$$

The several methods of finding the longitude differ in the modes of finding and comparing the two times, or the two hour-angles.

241. ***To find the longitude of a place by a portable chronometer regulated to Greenwich time.***

Solution. The correction and rate of the chronometer are supposed to have been found by suitable observations at a place whose longitude is known. Let the chronometer be transported to the place whose longitude is required; and let an observation suitable for finding the hour-angle of a heavenly body, or the local time, be made, and the time noted by the chronometer, or by a watch compared with it.

There are then two parts of the process to be pursued: 1st, from the noted time to find the Greenwich time (mean, apparent, or sidereal), or the hour-angle of the body, as may be deemed most convenient. 2d, from the observations, to find the corresponding local time, or hour-angle. Subtracting the local time, or hour-angle, from the Greenwich time, or hour-angle, will give the longitude.

242. 1st. *To find the Greenwich time, or hour-angle, of the body observed*, apply to the noted time the reduction of the watch time to chronometer time, $C - W$ (if a watch has been used) and the chronometer correction, c', reduced to the date of observation (Art. 219).

The result is, the Greenwich time; and will be *mean* or *sidereal*, according as the chronometer is regulated to mean or sidereal time.* If it is sidereal time, it will be necessary to reduce it to mean time (Art. 143), except when a fixed star has been observed, so as to take from the Almanac the quantities which will be required.

If, now, the Greenwich *hour-angle* of the body observed is desired:

If the case of the sun, reduce the Greenwich mean time to apparent time, by applying the equation of time.

If some other body has been observed, reduce the Greenwich mean time to sidereal time by adding the right ascension of the mean sun; and thence find the hour-angle of the body by subtracting its right ascension. Or, if a sidereal chronometer has been used, from the Greenwich sidereal time subtract the right ascension of the body.

Attention to the signs will give the hour-angle thus obtained, + if toward the *west*, − if toward the *east*.

243. The Greenwich time, or hour-angle, is affected by the error of the chronometer correction, which consists, 1st, of the error in its original determination, which includes any error of the assumed longitude of the place of rating; 2d, of the error arising from an erroneous rate. This last error is

* For observations of stars, a sidereal chronometer is most convenient.

cumulative, increasing with the number of days from the date, when the correction of the chronometer was found from observations.

244. The chronometer correction for the date of observation can be derived from subsequent as well as from prior determinations of it and its daily change. In finding the longitude of a place on shore, or of a shoal, both values should be obtained, when practicable, and combined by giving weights to each inversely proportional to its interval of time from the original determination. Thus, if c' and c'' are two such chronometer corrections, the first brought forward t' days, the second carried back t'' days, we may take as the mean value.*

$$\frac{t''\,c' + t'\,c''}{t' + t''},$$

or, in a form more convenient for computation,

$$c' + \frac{t'\,(c'' - c')}{t' + t''}.$$

For example, suppose that on January 17, the chronometer correction brought forward from January 1, is $-\ 18^m\ 56^s.5$, and reduced back from January 25, is $-\ 19^m\ 3^s.4$; the value by the above formula will be

$$-\ 18^m\ 56^s.5 + \frac{16 \times -\ 6^s.9}{24} = -\ 19^m\ 1^s.1.$$

Two longitudes may be combined in a similar way.

245. 2d. *To find the hour-angle of the body, and thence the local time.*

First Method. (Art. 180. By *single altitudes.*) Observe in quick succession several altitudes of the heavenly body, noting the time of each by the chronometer, or by a watch compared with it.

Take the mean of the noted times, and from it find the Greenwich mean time; for which take from the Almanac the declination of the body, its semidiameter and horizontal parallax when sensible, as well as the quantities required for finding the Greenwich hour-angle. (Art. 242.)

* This assumes that c' and c'' are derived from two chronometer corrections of equal weight, and consequently that the longitudes used in finding them are equally reliable. This may not be the case if the chronometer corrections were found from observations at two different places.

The student is referred to Chauvenet's "Astronomy," Vol. I, for the methods of allowing for changes in the rates and combining the results of several chronometers.

Take the mean of the readings of the instrument, with which the altitudes were measured, and from it find the *true* altitude of the center of the body. (Art. 153.) With this and the known, or assumed, latitude of the place find the *local hour-angle* of the body by Art. 180.

This hour-angle, which for the sun is the *local apparent time*, subtracted from the corresponding Greenwich hour-angle already found, will give the longitude.

Or, the *local mean time* may be found from it, for the sun, by applying the equation of time; for other bodies, by adding the right ascension of the body, which will give the *local sidereal time*, and subtracting the right ascension of the mean sun (Art. 148): and the local time subtracted from the corresponding Greenwich time will give the longitude.

246. On shore it is best to use an artificial horizon, even when a sea-horizon can be had, and for precise observations, stars in preference to the sun.

At sea the sun is most conveniently used; but altitudes of the moon and bright stars can be employed when the sun is not available. The chief difficulty is the obscurity of the sea-horizon at night. During twilight, however, or in a bright moonlight, it is often distinct and well defined.

247. The most favorable position of the body for finding its hour-angle from its altitude is, as previously stated, when it is nearest the prime vertical; provided its altitude is not so small as to involve to too great an extent the uncertainty of refraction; and, observed on shore, is within the limits* of the instruments employed.

On shore the time and circumstances most favorable for observations can generally be selected. At sea long continuance of bad weather may render poor observations, made under unfavorable circumstances, the only ones available.

While, then, it is not well to use for finding the time an altitude less than 10°, or of an object whose azimuth is less than 45° or more than 135°, it may sometimes be necessary to exceed these limits.

248. When the declination and latitude are nearly the same, the body is nearest the prime vertical but a short time before and after its meridian passage, so that a very great altitude may be used. Thus in Lat. 20° N., the sun, when its declination is 19° 55′ N. or 20° 5′ N., is nearest the prime vertical within 22^m of noon at an altitude of nearly 85°; and the local

* For a sextant and artificial horizon, between 20° and 60°.

time can be as accurately obtained from an latitude of 89°, 4^m from noon, and about 5° in azimuth from the prime vertical, as from an altitude of 30°, provided the assumed latitude can be depended on within 2′. Nearer noon, the rapid change of the sun's azimuth, averaging 10° in 1^m, would make it difficult to observe the altitude with sufficient precision.

249. The local time or hour-angle is affected by errors in the altitude and in the assumed latitude. (Arts. 184, 186.) When several observations have been made in rapid succession, the effect of a supposed error of 1′ in the altitude * may be found by dividing the difference of two of the noted times by the difference, in *minutes*, of the corresponding altitudes.

In a similar way we may find the change of altitude in 1^m of time by dividing the difference of two altitudes by the difference in *minutes* of the corresponding times. The maximum change of altitude in 1^m is 15′; when $L = 0$ and $d = 0$. The more rapid the change of altitude, the less will errors of altitude affect the result.

To ascertain the effect of an error of 1′ in the assumed latitude, † the local times or hour-angles may be computed separately for two latitudes differing 10′, or 20′, from each other, and the difference of these times divided by 10′, or 20′. At sea the latitude by account is used, either brought forward to the time of observation from a preceding, or carried back from a subsequent, determination. It may be very largely in error, especially in uncertain currents, or after running several days without observations.

A small error may also result from the assumption that the mean of the instrumental readings corresponds to the mean of the noted times. The reduction of the mean of the altitudes to the mean of the times can be found,‡ but it can be avoided

* Differentiating equation (71)

$$\sin h = \sin L \sin d + \cos L \cos d \cos t,$$

regarding h and t as variables, we have

$$\cos h \, d h = -\cos L \cos d \sin t \, d t$$

but $\cos d \sin t = \cos h \sin Z$ SPH. TRIG.

whence $$d t = -\frac{d h}{15 \cos L \sin Z},$$

which is a minimum when $Z = \pm 90°$, and incalculable when $Z = 0°$ or 180°.

† From (102) we find

$$d t = -\frac{d L}{15 \cos L \tan Z},$$

which is 0, when $Z = \pm 90°$, and also incalculable when $Z = 0$ or 180°.

‡ Chauvenet's "Astronomy, Vol. I."

by limiting the series of observations, which are combined together, to so brief a period that the error becomes insensible; or, when the body is near the meridian in azimuth, by reducing each observation by itself. This last case, however, should be avoided in this problem.

250. At sea it is usual to reduce longitudes obtained from day observations to noon by allowing for the run of the vessel in the interval, and for currents when known. Those from night observations are recorded for the time of observation.

251. *Second Method.* Altitudes in the forenoon and in the afternoon, or on different sides of the meridian, are preferable to single altitudes for finding the local time, for the reasons already stated in Article 227. The longitudes can be found from each set separately, and then combined.

At sea the longitudes derived from each can be reduced to noon, and the mean of the two taken as the true longitude; or, if the difference can be regarded as due to currents, the longitude at noon can be found by interpolating for the elapsed time. It is desirable that the observations should be made at nearly equal intervals from noon.

Longitudes by A.M. and P.M. observations are desirable whenever practicable.

252. *Third Method.* (By *equal* altitudes.) Let equal altitudes of a heavenly body be observed east and west of the meridian (Art. 228) and the times noted as in other observations; and the mean of the watch-times in each set, if a watch is used, reduced to chronometer time. If both sets have been observed at the same place, and the declination of the body has not changed, the mean of the two times will be the chronometer time of its meridian transit.

If the declination has changed in the interval, as is ordinarily the case with the sun, moon, or a planet, the correction for such change, found by the methods of Art. 241, should be applied.

Applying then the chronometer correction, we have the corresponding Greenwich time, which will be mean or sidereal as the time to which the chronometer is regulated.

EXAMPLE 1: January 1, 1918, in Lat. 40° 00′ N., Lon. 39° 45′ W., by D. R. the sextant altitude of Jupiter, was 31° 51′ 20″, planet E. of meridian, I. C. 0′ 00″, height of eye, 30 feet, W. T. of observation $5^h\ 01^m\ 07^s$ P.M., C − W $2^h\ 38^m\ 52^s$, chronometer fast $0^m\ 23^s$. Find the longitude.

Time

	h	m	s
W.	5	01	07
C − W	2	38	52
C	7	39	59
c. c.		−	23
G. M. T. Jan 1	7	39	36
	= $7^h.66$		

	h	m	s
G. M. T.	7	39	36
R. A. M. S.	18	42	33.9
G. S. T.	2	22	09.9

Alt.

	°	′	″
Corr.	31	51	20
	−	6	56
h =	31	44	24

Corr.

	′	″
Dip	−5	22
ref.	−1	34
Corr.	−6	56

R. A. and Dec.

	h	m	s
	4	02	29
Corr.			− 6.7
R. A. =	4	02	22.3

H. D. 0ᵈ.875
G. M. T. $7^h.66$
Corr. −6.702

	°	′
Dec.	+19	54.3
Corr.		−.2
Dec. N.	19	54.1
p =	70	05.9

H. D. −0′.029
G. M. T. $7^h.66$
Corr. −0.223

	h	m	s
R. A. M. S.	18	41	18.4
Corr. G. M. T.		+1	15.5
	18	42	33.9

	°	′	″		
h =	31	44	24		
L =	40	00	00	sec	0.11575
p =	70	05	54	cosec	0.02674
2 |	141	50	18		
s =	70	55	09	cos	9.51442
h	31	44	24		
s − *h*	39	10	45	sin	9.80055

	h	m	s		
t = −	4	19	00.2	= hav	9.45746
R. A. Jup.	4	02	22.3		
L. S. T.	23	43	22.1		
G. S. T.	2	22	09.9		
Lon.	2	38	47.8 W.		
	= 39°	41′	57″ W.		

Finding from this, by the method in Art. 242, the *Greenwich hour-angle* of the body (which in the case of the sun is the Greenwich apparent time), we have the longitude, if the first observation was *east* of the meridian, as the corresponding *local hour-angle* is then 0. But if the first observation was west of the meridian, the local hour-angle is 12^h, and must be subtracted.

This method should be used on shore, when practicable, in preference to either of the preceding.

253. Equal altitudes of the sun can be conveniently used at sea when the sun passes the meridian near the zenith; that is, when its declination and the latitude are nearly the same. Altitudes very near noon are then available for finding the time (Art. 248), and equal altitudes can be observed with only a short interval. In the example of Art. 248, an interval of eight minutes would have been sufficient.

If the vessel does not change her position in the interval, the middle time corresponds to apparent noon; as the change of declination may be neglected, unless the interval between the observations is so great as to require it.

254. If the longitude only has changed, the middle time corresponds to apparent noon at the middle meridian, and will give the longitude of that meridian. This will be the longitude at noon, if the speed of the vessel has been uniform. But if it has not, subtracting half the change of longitude, when the true course is *west*, or adding it when the course is *east*, will give the longitude of the place where the first altitude was observed. This can then be reduced to noon by allowing for the run of the vessel.

If the change of longitude is west, the sun arrives at the corresponding latitude of the afternoon later than it would do if observed at the same place as in the forenoon; if the change is east, it arrives earlier; and the difference is the time of the sun's passing from the one meridian to the other; that is, the difference of longitude expressed in time.

If, then, $2t$ is the elapsed apparent time,

$\Delta\lambda$, the change of longitude (+ when west),

the hour-angle of the sun at each observation is $t - \frac{1}{2}\Delta\lambda$; and (130) becomes

$$\Delta T_0 = -\frac{\Delta_h\, d\, t \tan L}{15 \sin (t - \frac{1}{2}\Delta\lambda)} + \frac{\Delta_h\, d\, t \tan d}{15 \tan (t - \frac{1}{2}\Delta\lambda)}. \quad (139)$$

But even when the elapsed time is so great that it is thought necessary to correct for the change of declination, $\Delta\lambda$ is never large enough to produce a change of 1^s.

If the latitude only has changed, the middle time requires correction for such a change, which can be deduced in a similar way to that for a change of declination in Art. 241. But, as in the fundamental formula (71),

$$\sin h = \sin L \sin d + \cos L \cos d \cos t,$$

L and d enter with the same function, they are interchangeable. If, then,

$\Delta_h L$ is the hourly change of latitude (+ toward the north and expressed in seconds), and

$\Delta' T_0$, the required correction,

we have from (130) and (132),

$$\Delta' T_0 = -\frac{\Delta_h L\, t \tan d}{15 \sin t} + \frac{\Delta_h L\, t \tan L}{15 \tan t}. \qquad (140)$$

and $$\Delta' T_0 = A\, \Delta_h L \tan d + B\, \Delta_h L \tan L, \qquad (141)$$

for which Chauvenet's tables can be used.

If both latitude and longitude have changed, for t in the denominators of (140), we may substitute $t - \frac{1}{2}\Delta\lambda$: but this at sea is a needless refinement.

The restriction of this method to a short interval between the observations depends upon the uncertainty of the run of the vessel, and consequent imperfect determination of $\Delta_h L$, the mean hourly change of latitude in the interval. If its error is supposed to be $\frac{1}{n}\Delta_h L$, the consequent error in $\Delta' T_0$ is $\frac{1}{n}\Delta' T_0$.

When equal altitudes near noon are practicable, a meridian altitude of the sun can ordinarily be taken for latitude, so that L will be sufficiently exact. Moreover, the latitude and longitude are both found for noon.

EXAMPLE: February 3, 1918, at Tampico, Mexico, in Lat. $22^\circ\ 15'\ 50''$ N., observed with a sextant and artificial horizon, equal altitudes of the sun in A.M. and P.M., I. C. $0'$, mean of A.M. chronometer times $3^h\ 31^m\ 26^s.925$, mean of P.M. chronometer times $10^h\ 07^m\ 40^s.893$ chronometer fast of G. M. T. $4^m\ 17^s.5$. Find the longitude. (Use Ephemeris.)

Time and Required Longitude

	h	m	s
C. T. (A.M.)	3	31	26.925
c. c.	−	4	17.5
G. M. T. (A.M.)	3	27	09.425
G. M. T. (P.M.)	10	03	23.393
G. M. T. (Mid.)	6	45	16.409
1st and 2d part		−	7.672
G. M. T. of app. noon	6	45	08.737
Eq. of t.	−	13	56.951
G. A. T. of app. noon or Lon. W.	6	31	11.786

Lon. = 97° 47′ 56″.79 W.

t

	h	m	s
C. T. (P.M.)	10	07	40.893
c. c.	−	4	17.5
G. M. T. (P.M.)	10	03	23.393
G. M. T. (A.M.)	3	27	09.425
$2t$ =	6	36	13.968
t =	3	18	06.984

t = 3ʰ.302

= 49° 31′ 45″

	°	′	″			
L =	22	15	50 N.	tan	9.61214	
d =	16	35	28 S.	tan		−9.47414
Δhd =	43″.983			log	1.64329	1.64329
t =	3ʰ.302			log	0.51878	0.51878
t =	49°	31′	45″	cosec	0.11877	
15 =				colog	8.82391	8.82391
− 1st part					+0.71689	
				2d part	−2.4613	−0.39117
				1st part	−5.2106	
				Corr.	−7.6719	

Eq. of T.

	m	s
	− 13	55.08
Corr.	−	1.871
	−13	56.951

(− to mean time)

H. D. (mean)

	°
	−0.277
G. M. T.	6ʰ.754
Corr.	−1ˢ.8708

	s
H. D. Eq. t., Feb. 3	0.281
H. D. Eq. t., Feb. 4	0.248
Change 24ʰ	0.033
Change $\frac{\text{G. M. T.}}{2}$	0.004
H. D., Feb. 3	0.281
H. D. Eq. t. (mean)	−0.277

	Dec. °	′	″	H. D. (mean) ″
	−16	40	24.5	+43.886
	+	4	56.4	G. M. T. 6ʰ.754
	S. 16	35	28.1	Corr. + 296.406

	″
H. D. dec. Feb. 3	+43.79
H. D. dec. Feb. 4	+44.51
Change 24ʰ	+ 0.72
Change 1ʰ	+ 0.030
Change G. M. T. 6ʰ.754	+ 0.1926
H. D., Feb. 3, G. M. N.	+43.79
Δhd	+43.983
Change $\frac{\text{G. M. T.}}{2}$	″ = 0.096
H. D., Feb. 3	43.790
D. H. dec. (mean)	+43.886

255. *Fourth Method.* (By *transits.*)

Observe the transits of the sun or a star across the threads of a well-adjusted transit instrument, noting the times. Reduce the mean of the noted times for semidiameter and errors of the instrument as in Art. 237; and thence find the Greenwich hour-angle of the body in the way described in Art. 242. This will be the longitude, if the upper culmination has been observed, as the local hour-angle is 0. If the lower culmination has been observed, the local hour-angle is 12^h.

This method can be used only on shore.

EXAMPLE 1: May 9, 1918 (Ast. day), in Lon. 44° 39′ E., observed the transit of Arcturus over the middle wire of the telescope, the time noted by a chronometer regulated to G. M. T., being $8^h\ 10^m\ 40^s.7$. Find the chronometer correction.

L. S. T. (R. A. *)	14^h	11^m	$58^s.4$
Lon., E.	-2	58	36
G. S. T.	11	13	22.4
R. A. M. S., $9^d\ 0^h$	-3	05	57.4
Sid. int. from 0^h	8	07	25.0
Reduction (Tab. II)		-1	19.9
G. M. T.	8	06	05.1
Chro. t.	8	10	40.7
Chro. fast		4	35.6

EXAMPLE 2: June 25, 1918, in Lon. 60° E., observed the transit of both limbs of the sun over the meridian wire of the telescope, noting the times by a chronometer. Find the error of the chronometer on G. M. T.

Transit of western limb	8^h	05^m	$02^s.5$
Transit of eastern limb	8	07	20.0
Chro. time, loc. app. noon	8	06	11.25
L. A. T., loc. app. noon	0^h	00^m	00^s
Eq. t.	+	2	12.95
L. M. T., loc. app. noon	0	02	12.95
Lon., E.	-4	00	00
G. M. T., loc. app. noon	8	02	12.95
Chro. time, loc. app. noon	8	06	11.25
Chro. fast		3	58.3

Lon. place	4^h	00^m	00^s E.
Lon. Wash.	5	08	15.8 W.
	9	08	15.8 E. of Washington

or W. A. T. = June 24, $14^h\ 51^m\ 42^s = 14^h.86$

Eq. of T., June 24 at Wash.	$+2^m$	$05^s.03$
Corr. $14^h.86$		+7.92
Eq. t. L. A. N.	2	12.95

CHAPTER XIII

SUMNER'S METHOD—LINES OF POSITION

CIRCLES OF EQUAL ALTITUDE.— (SUMNER'S METHOD)

256. SUPPOSE that at a given instant the sun, or any other heavenly body, is in the zenith of the place M (Fig. 43), on the earth; and let A A′ A″ be a small circle described from M as a pole. The zenith distance of the body will be the same at all places on this small circle, namely, the arc M A; for if the representation is transferred to the celestial sphere, or projected on the celestial sphere from the center as the projecting point,

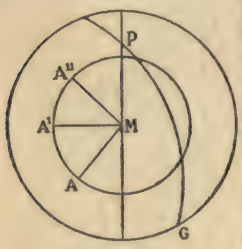

FIGURE 43

M will be the place of the sun, or other body, and the circle A A′ A″ will pass through the zeniths of all places on the terrestrial circle, and

M A, M A′, etc., will be equal zenith distances.

The altitude of the body will also be the same at all places on the terrestrial circle A A′ A″; hence such a circle is called a *circle of equal altitude.*

It is evident that this circle will be smaller the greater the altitude of the body.

257. The latitude of M is equal to the declination of the body, and its longitude is the Greenwich hour-angle of the body; which, in the case of the sun, is the Greenwich apparent time, or 24^h — that apparent time, according as the time is less or greater than 12^h. This is evident from the diagram, in which, regarded as on the celestial sphere,

P M is the celestial meridian of the place, whose zenith is M, and its co-latitude; and also the declination circle, and co-declination, of the body M;

and if P G is the celestial meridian of Greenwich, G P M is, at the same time, the longitude of the place, and the Greenwich hour-angle of the body.

If, then, the Greenwich time is known, the position of M may be found and marked on an artificial globe.

258. If, moreover, the altitude of the body is measured, and a small circle is described on the globe about M as a pole, with the complement of the altitude as the polar radius, the position of the observer will be at some point of this circle. His position, then, is just as well determined as if he knew his latitude alone, or his longitude alone; since a knowledge of only one of these elements simply determines his position to be on a particular circle, without fixing upon any point of that circle.

As, however, he may be presumed to know his latitude and longitude approximately, he will know that his position is within a limited portion of this circle. Such portion only he need consider. It is commonly called a *line of position.*

259. The direction of this line at any point is at right angles with the direction of the body, or the *line of bearing*, as it is called; for the polar radius M A is perpendicular to the circle A A′ A″ at A, A′, A″, and every other point of the circle.

The geographical position of a heavenly body.— To understand the theory of the position lines, as obtained from the observed altitude of a celestial body and the G. M. T., it is necessary to understand what is meant by the geographical position. If a straight line be drawn from the center of the celestial body perpendicular to the earth's surface, the point where the line intersects the earth's surface is the geographical position of the body, and the heavenly body is in the zenith.

260. Suppose that in Lon. 25° W. on May 21, 1918, the sun's declination is 20° N., at L. A. T., $9^h\ 20^m$ A.M. or Greenwich apparent time, May 20, 1918, 23^h, and that a vessel in the N. Atlantic observed a true altitude of the sun of 40°. The geographical position will then be (Fig. 44) at a point in Africa having a latitude of 20° N., and longitude of 15° E., — at *A*. And the radius of the "circle of position," or the distance from *A* to *C* — will be 50°, or 3000 miles.

A second observation is then made three hours later, when the sun's true altitude was 65°. The geographical position will then be at *B*, Lat. 20° N., Lon. 30° W., and the radius of the circle of position *B C* will be 25°, or 1500 miles, *C* being the vessel's position.

261. Artificial globes are constructed on so small a scale that the projection of a circle of equal altitude on a globe would give only a rough determination. But the projection of a

limited portion may be made upon a chart by finding as many points of the curve as may be necessary, and, having plotted them upon the chart, tracing the curve through them. The portion required is usually so limited that, when the altitude of the body is not very great, it may be regarded as a straight

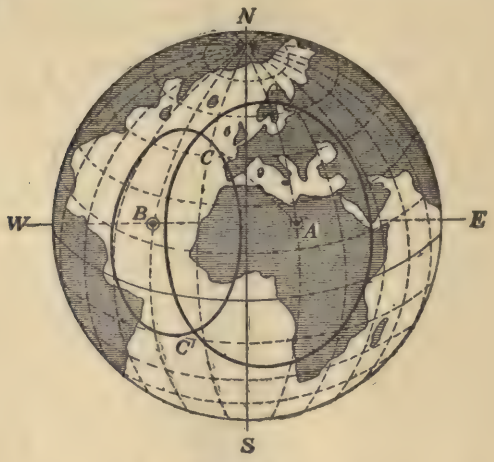

FIGURE 44

line; and hence two points suffice. With high altitudes, three points, or if the body is very near the zenith, four may be necessary, and even the entire circle may be required.

262. ***From an altitude of a heavenly body to find the line of position of the observer, the Greenwich time of the observation being known.***

Solution. From the given altitude, and assumed latitudes L_1, L_2, L_3, etc., differing but little from the supposed latitude, find the corresponding local times (Art. 180), and thence, by the Greenwich time, the longitudes λ_1, λ_2, λ_3, etc. Thus we shall have the several points, whose positions are conveniently designated as (L_1, λ_1), (L_2, λ_2), (L_3, λ_3), etc.

It facilitates the computation to assume latitudes differing 10′ or 20′, as the $\frac{1}{2}$ sums and remainders differ 5′ or 10′, and only one of each need be written.

Or, from the Greenwich time and assumed longitudes, λ_1, λ_2, λ_3, etc., find the corresponding local times (Art. 119), and

thence the hour-angles of the body (Art. 145, 146). With these and the observed latitude, find the corresponding altitudes, L_1, L_2, L_3, etc. (Art. 193).

This is more convenient than the preceding method, when the body is near the meridian.

In either mode the computation may be arranged so that the like quantities in the several sets shall be in the same line, and taken out at the same opening of the tables.

The several points may then be plotted on a chart, each by its latitude and longitude, and a line traced through them, which will be the required *line of position*. Two points connected by a straight line are sufficient, unless the altitude is very great, or the points widely distant.

Thus in (Fig. 45), let A and B be two such points plotted respectively on the parallels of latitude L_1, L_2, and each in its proper longitude; A B is the *line of position*, and the place of observation is at some point of A B, or AB produced. This is *all* which can be determined from an observed altitude, unless either the latitude, or the longitude, is definitely known. And as these are both uncertain at sea, except at the time when found directly by observation, the position of the ship found from a single altitude, or set of altitudes, is a *line*, of greater or less extent as the latitude, or the longitude, is more or less accurately known.

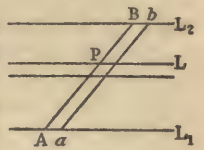

FIGURE 45

In uncertain currents, or when no observations have been had for several days, the extent of this line may be very great. Yet, if it is parallel to the coast, it assures the navigator of his distance from land; if directed toward some point of the coast, it gives the bearing of that point.

263. If there is uncertainty in the altitude, for instance of $3'$, the line of position having been computed and plotted, parallels to it on each side may be drawn at the distance of $3'$.

So, also, if there is uncertainty in the Greenwich time, parallels may be drawn at a distance in *longitude* equal to the amount of uncertainty.

In either case the position of the vessel is within the inclosed belt.

In Fig. 45, $a\,b$ is such a parallel to the line of position A B, its perpendicular distance from it measuring a difference of altitude; the distance A a on a parallel of latitude measuring a difference of longitude.

264. Since the line of position is at right angles with the direction of the body (Art. 259), the nearer the body is to the meridian in azimuth, the more nearly the line of position coincides with a parallel of latitude; and thus a position of the body near the meridian is favorable for finding the latitude from an observed altitude, and not the longitude.

So also, the nearer the body is to the prime vertical, the more nearly the line of position coincides with a meridian, and the less does any error in the assumed latitude affect the longitude computed from an observed altitude. So that, if the body is on the prime vertical, a very large error in the assumed latitude will not sensibly affect the result. Such a position of the body is, then, the most favorable for finding the longitude from an observed altitude.

These conclusions have been previously stated, drawn from analytical considerations.

265. Two or more points of a line of position as (L_1, λ_1), (L_2, λ_2), etc., having been determined by Art. 262, if the true latitude, L, be subsequently found, the corresponding longitude, λ, may be obtained by interpolation.

Or, the place of the vessel may be found graphically upon the chart, by drawing a parallel in the latitude, L, and taking its intersection P, with the line of position A B.

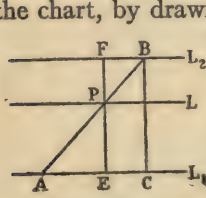

FIGURE 46

So also, if the true longitude, λ, is subsequently found, the corresponding latitude, L, may be obtained by interpolation; or, a meridian E F may be drawn in the longitude, λ, which will intersect the line of position in P, the place of the vessel.

If there is uncertainty in either of these elements, two parallels of latitude (as in Fig. 45), or two meridians, may be drawn at a distance apart equal to the uncertainty.

As altitudes, latitudes, and longitudes are never found at sea with much precision, and may under unfavorable circumstances be largely in error, the position of the vessel on the chart is not properly a *point*, but a *belt*, more or less limited according to the accuracy of the elements from which it has been formed.

266. There are two methods of determining the line of position, first the *method of chords* where for short distances the chord obtained is coincident with the included arc of the circle: this may be defined by its two points A' A'' as

$$A' \left\{ \begin{matrix} 18^\circ\ 10' \text{ N.} \\ 2^\circ\ 47' \text{ E.} \end{matrix} \right. \qquad A'' \left\{ \begin{matrix} 19^\circ\ 30' \text{ N.} \\ 2^\circ\ 37' \text{ E.} \end{matrix} \right.$$

and second by the *method of tangents*, the true azimuth of the body for the instant of observation having been taken from the azimuth tables, it is defined by its one position, point A, and direction thus:

$$\text{Pos. } A. \left\{ \begin{matrix} 35^\circ\ 00' \text{ S.} \\ 15^\circ\ 10' \text{ W.} \end{matrix} \right\} \text{Az.} = \text{N. } 67^\circ\ 10' \text{ E. from Az. Tables.}$$

267. In Fig. 46, if A is the position (L_1, λ_1),
B, the position (L_2, λ_2),
both near P, the true position, whose latitude is L, and longitude is λ;
we have, by interpolation

$$\left. \begin{aligned} \Delta\lambda &= \Delta L \frac{\lambda_2 - \lambda_1}{L_2 - L_1} \\ \text{and} \qquad \lambda &= \lambda_1 + \Delta\lambda \end{aligned} \right\} \qquad (142)$$

as the formulas for finding λ, the longitude of the true position, when its latitude, L, is known.

268. The longitude factor, of a line of position represented by the letter F, is the change of longitude due to 1′ change in latitude. By the chord method it is found as follows:

$F = \dfrac{\lambda_2 - \lambda_1}{L_2 - L_1} = \dfrac{\Delta\lambda}{\Delta L}$, where ΔL is a change of latitude due to a change of longitude and vice versa. The longitude factor, F, is given in table 47, Bowditch, at convenient intervals of latitude and azimuth. The quantities tabulated are computed from the formula

$$F = \text{sec. Lat.} \times \text{cot. Az.}$$

When a time sight is solved with a D. R. latitude, the resulting longitude is true if the latitude is correct. This table by setting forth the number of minutes of longitude due to each minute of error in latitude, gives the means of finding the correction to the longitude for any error that may subsequently be found in the latitude used in the computation.

Regarding the azimuth of the observed celestial body as less than 90° and as measured from either the North or the South point of the horizon towards East or West, the rule for determining whether the correction in longitude is to be applied to

the eastward or to the westward is as follows: If the change in latitude is of the same name as the first letter of the bearing, the change in longitude is of the contrary name to that of the second letter, and vice versa.

Thus, if the body bears S. 45° E. and the change in latitude is southward, the change in longitude is westward; and, if the change in latitude is northward, the change in longitude is eastward.

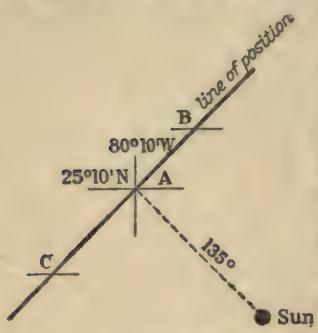

FIGURE 47

269. This may be illustrated clearly so that the rule need not be remembered. Draw a line through the vessel's position on the chart perpendicular to the bearing obtained from the azimuth tables at the instant of observation. Suppose the azimuth is S. 45° E., then the position line will run N.E. – S.W. Let *A* be the position of vessel (Lat. 25° 10′ N., Lon. 80° 10′ W.) by D. R. If the D. R. latitude is wrong and the true latitude is found to be farther north at *B*, when a line is drawn through this latitude the true longitude is more to the eastward, if the true latitude is to the southward at *C*, the longitude is more to the westward.

270. Suppose the sun bears east or west that is near the prime vertical, a considerable change in latitude to the northward or southward has slight or no effect on the longitude, hence the reason that time sights for longitude are taken when the body is on or close to the prime vertical.

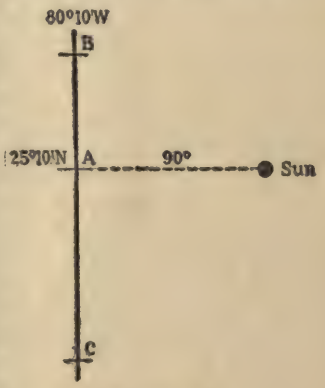

FIGURE 48

271. Suppose the sun bears S. × E., the position line will be about 11° from an east-west direction, and it is clear that a small error in the latitude will make a great error in the longitude, therefore this shows why a time sight for longitude should not be observed when the body bears north or south.

We also have from Fig. 46

$$\left.\begin{aligned} \Delta L &= \Delta\lambda \frac{L_2 - L_1}{\lambda_2 - \lambda_1} \\ \text{and} \qquad L &= L_1 + \Delta L \end{aligned}\right\} \tag{143}$$

as the formulas for finding L, when λ is given. The several differences are most conveniently expressed in minutes of arc, or, in the case of longitudes, in seconds of time. The local times may be used instead of the longitudes and interpolated in the same way.

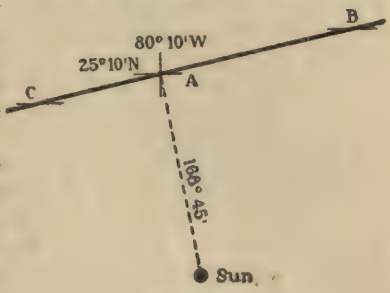

FIGURE 49

The latitude factor of a line of position is represented by $f = \frac{1}{F}$ or the change in latitude due to a change of 1′ of longitude

$$\frac{1}{F} = \frac{L_2 - L_1}{\lambda_2 - \lambda_1} = \frac{\Delta L}{\Delta \lambda}.$$

272. The latitude factor, f, is given in table 48, Bowditch, at convenient intervals of latitude and azimuth. The quantities tabulated are the reciprocals of the values of the longitude factor, and computed from the formula—

$$f = \frac{1}{F} = \frac{1}{\text{sec. Lat.} \times \text{cot. Az.}} = \text{cos. Lat.} \times \text{tan. Az.}$$

When an ex-meridian sight is solved with a longitude afterwards found in error, this table, by setting forth the number of minutes of latitude due to each 1′ of error in longitude, gives the means of finding the correction in the latitude for the amount of error in the longitude used in the computation.

Regarding the azimuth of the observed celestial body as less than 90° and as measured from either the North or the South point of the horizon towards East or West, the rule for determining whether the correction in latitude is to be applied to the northward or to the southward is as follows: If the change in longitude is of the same name as the second letter of the bearing, the change in latitude is of the contrary name to the first letter, and vice versa. Thus, if the body bears S. 14° E.

and the change in longitude is westward, the change in latitude is southward; if the change in longitude is eastward, the change in latitude is northward.

273. ***To find from a line of position the azimuth of the body observed.***

Solution. We have the positions (L_1, λ_1), (L_2, λ_2), or the latitudes and longitudes of two points, from which the azimuth, or course of the line of position, can be found by *middle latitude sailing.*

Adding or subtracting 90°, according as the azimuth of the body is greater or less, gives the azimuth required.

Or, a perpendicular to the line of position may be drawn upon the chart, and the angle which it makes with a meridian may be measured with a protractor. The azimuth may thus be found to the nearest 1°. The better way is to take the azimuth from the tables and draw a line from the one determined position in the direction of the sun's bearing, then draw the line of position perpendicular so that it passes through the determined position.

274. ***To find the position of the observer from two altitudes of the same or different bodies, the Greenwich time being known.***

Solution. Find the line of position from each. If the lines are plotted on the chart, their intersection gives the position required. This intersection may also be readily found by computation and the first example will be the most simple, that is, where the line runs true N. and S. when the observed body is on the prime vertical, and the other line runs true E. and W. when the observed body is on the meridian.

EXAMPLE: May 1, 1918, in Lat. 24° 56′ S., Lon. 60° 00′ W., by D. R., observed simultaneous altitudes of Regulus (α Leonis), on the meridian bearing N., and of Sirius (α Can. Maj.) on the prime vertical W. of meridian. The sextant altitude of Regulus was 52° 46′ 42″ and of Sirius 42° 28′ 00″, I. C. − 2′ 30″, height of eye 30 feet, W. T., $7^h\ 28^m\ 00^s$ P.M., chronometer fast $3^m\ 20^s$. Find the vessel's position. (From Almanac, Regulus, transits at $7^h\ 28^m$ P.M.)

Time				Regulus				Corr. Alt.			Sirius				Corr. Alt.			R. A. M. S.			
	h	*m*	*s*		°	′	″		′	″		°	′	″		′	″		*h*	*m*	*s*
W.	7	28	00	*'s h_0	52	46	40	I. C.	−2	30	*'s h_0	42	28	00	I. C.	−2	30		2	34	25.0
C − W	4	03	20	Corr.	−	8	36	Dip	−5	22	Corr.	−	8	55	Dip	−5	22		+	1	53.0
C	11	31	20	*'s *h*	52	38	04 N.	*r*	−	44	*'s *h*	42	19	05	*r*.	−1	03		2	36	18
c. c.	−	3	20	*'s *z*	37	21	56 S.	Corr.	−8	36					Corr.	−8	55				
G. M. T.	11	28	00	*'s *d*	12	21	54 N.														
May 1				Lat. =	25	00	02 S.														

	h	*m*	*s*		°	′
Regulus, R. A.	10	04	02.9	Dec. N.	12	21.9
Sirius, R. A.	6	41	32.9	Dec. S.	16	36.4
				p =	73	23.6

	°	′	″		
h =	42	19	05		
L =	25	00	02	sec	0.04272
p =	73	23	36	cosec	0.01851
2*s*	140	42	43		
s	70	21	21	cos	9.52657
h	42	19	05		
s − *h*	28	02	16	sin	9.67214

	h	*m*	*s*
t = hav 9.25994 =	3	21	59.4
R. A. Sirius	6	41	32.9
L S. T.	10	03	32.3
R. A. M. S.	2	36	18.0
L. M. T.	7	27	14.3
G. M. T.	11	28	00.0
Lon.	4	00	45.7 W.
	60°	11	26″ W.

275. The more nearly perpendicular the lines of position are to each other, the better is the determination of their intersection. Hence, the nearer the difference of azimuths of the body or bodies at the two observations is to 90°, the better is the determination of position from double altitudes.

If the azimuths are the same, or differ 180°, the two lines of position coincide in direction, and there is no intersection. In this case the great circle joining the two bodies, or the two positions of the same body, is an azimuth circle, and passes through the zenith. An approach to this condition is generally to be avoided. Still, however, if the two bodies, or positions of the same body, are near the meridian, the lines of position nearly coincide with a parallel of latitude. The latitude is then well determined, but not the longitude. If the two bodies, or positions of the same body, are near the prime vertical, the lines of position more nearly coincide with a meridian, and the longitude is well determined; but not the latitude.

When the difference of azimuths is small, the intersection of the two lines cannot be definitely found by the projection of the lines upon a chart.

276. To determine the intersection of two Sumner lines running at angles with parallels and meridians when the position points having a common latitude are known, one for each Sumner line.

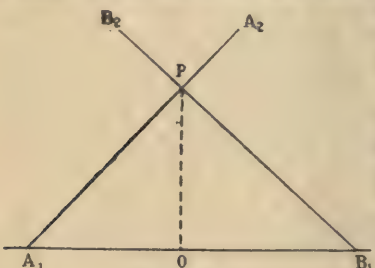

FIGURE 50

Two Sumner lines with position points on the same parallel as when simultaneous observations of two bodies for finding time, the azimuths of each being between 45° and 135° or between 225° or 315°, also when a line for a time sight is found in conjunction with one from a ϕ'' ϕ' sight, the latitude corrected for the run of the vessel being used in the time sight.

There are two cases,

Case I. When one line runs in a NE.–SW. direction, and the other in a NW.–SE. direction, as shown in Fig. 50.

Case II. When both run in a NE.–SW., or both in a NW.–SE. direction, as shown in Fig. 51.

Let A_1 A_2 be the first line, its longitude factor found from Table 47. Then if P is the point of intersection $\Delta L = P O$ and the corresponding difference of longitude from A is $\Delta \lambda = A_1 O$ $\because \Delta \lambda = \Delta L \times F$ B_1B_2 = second line, its factor is F_1, and in same way $B_1 O = \Delta \lambda_1$ and $\Delta \lambda_1 = \Delta L \times F_1$.

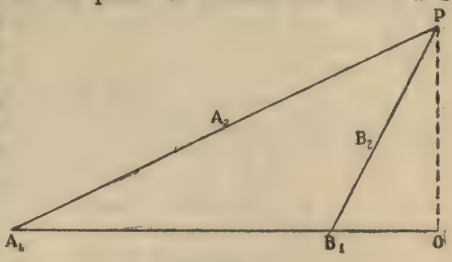

FIGURE 51

In both figures A_1 B_1 = difference of longitude of both lines for one assumed latitude. In Fig. (51)

$$A_1 B_1 = A_1 O - B_1 O = P O(F - F_1)$$

or $$A_1 O - B_1 O = \Delta L(F - F_1)$$

but $$\Delta \lambda = A_1 O = \Delta L \times F$$

and $$\Delta \lambda_1 = B_1 O = \Delta L \times F_1$$

Where lines run same way and are alike both $\Delta \lambda$ and $\Delta \lambda_1$ are applied in the same direction both either E. or either W. so as to make the longitudes the same.

Having the name of correction in longitude for either line and the direction, the name of the correction in latitude is known. In Fig. (50),

$$A_1 B_1 = A_1 O + O B_1 = P O\,(F + F_1) = \Delta L(F + F_1)$$

and $\Delta \lambda = A_1 O = \Delta L \times F$ and $\Delta \lambda_1 = O B_1 = \Delta L \times F_1$

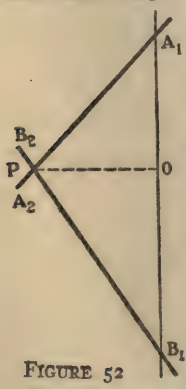

FIGURE 52

Here the lines are unlike and $\Delta \lambda$ and $\Delta \lambda_1$ are applied opposite, westward to the more eastward longitude and eastward to the more westward longitude so as to make both longitudes the same. Knowing the correction in longitude for either line and the direction, the name of corresponding latitude is found. Apply correction in latitude to latitude of parallel used as origin to find latitude of vessel's position.

277. To find the intersection of two Sumner lines running at an angle to meridians and parallels when the position points having the same longitude are known, one for each line.

Two Sumner lines with position points on a common meridian as the case of two simultaneous observations worked for lati-

tude by the ϕ'' ϕ' sight, or in case of a position line combined with one worked as a time sight, the computed longitude from the time sight after correction for run of vessel being used in the ϕ'' ϕ' sight.

The latitude factor Table 48 is used.

Let $A_1 A_2, B_1 B_2$ (Fig. 53) be two lines obtained by using the same assumed longitudes or let $A_1 A_2$ be the first line moved for run of vessel, $A_1 A_2$ being coördinates of first line at second observation, the longitudes of $A_1 A_2$ being used to work the sights for line $B_1 B_2$. Let L and L_1 be the latitudes of $A_1 A_2$, L' and L_1' of $B_1 B_2$; $A_1 A_2$ = first line and f = latitude factor then if P = point of intersection, $\Delta\lambda = P O$ and ΔL from parallel $A_1 = A_1 O$ but $A_1 O = P O \times f \therefore \Delta L_1 = \Delta\lambda \times f_1$.

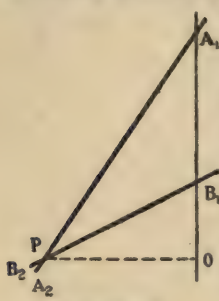

FIGURE 53

In both figures $A_1 B_1$ = different latitude of position points of the Sumner lines on the common meridian $A_1 B_1$.

In (Fig. 53) $A_1 B_1 = A_1 O - B_1 O = P O (f - f_1)$ or difference latitude $A_1 B_1 = \Delta\lambda (f - f_1)$ then $A_1 O = \Delta L = \Delta\lambda \times f$ and $O B_1 = \Delta L_1 = \Delta\lambda \times f_1$.

In this figure where lines run alike both corrections to the latitude ΔL and ΔL_1 are applied the same way either to northward or southward so as to make the latitudes the same.

Having the name of the correction in latitude for either line the name of the correction for longitude for the same line is also known. In Fig. (52)

$A_1 B_1 = A_1 O + O B_1 = P O (f + f_1)$, or known difference of latitude $A_1 B_1 = \Delta\lambda (f + f_1)$,

then

$$A_1 O = \Delta L = \Delta\lambda \times f$$

and

$$O B_1 = \Delta L_1 = \Delta\lambda \times f_1$$

Here the line are unlike, the corrections for latitude are applied in opposite directions northward to the more southward latitude and southward to the more northward latitude so as to make both latitudes the same. The name of correction in longitude is found by the rule. The correction in longitude applied with its proper sign to the longitude of the same meridian will give the longitude of the vessel's position.

EXAMPLE 1: January 15, 1918, in Lat. 40° N., Lon. 39° 40′ W., by D. R. observed with a sextant simultaneous altitudes of the planets, Jupiter bearing northeastward, and the planet Venus bearing northwestward, observed altitude of Jupiter 48° 24′ 20″; observed altitude of Venus 23° 57′ 30″; I. C. + 1′ 30″, height of eye 30 feet, W. T. of observation $5^h\ 30^m\ 00^s$ P.M., C − W $2^h\ 39^m\ 52^s$, chronometer fast $0^m\ 52^s$. Find the vessel's position, assuming latitudes 39° 30′ N., and 39° 50′ N.

Time				Jupiter						Venus					
				Alt.			R. A. and Dec.				Alt.			R. A. and Dec		
	h	*m*	*s*	°	′	″	*h*	*m*	*s*	*H. D.*	°	′	″	*h*	*m*	*s*	*H. D.*
W.	5	30	00	*'s *h*₀	48	24	20	R. A.	3	58	54	−0ˢ.375	*'s *h*₀	23	57	30	R. A.	21	57	03	+0ˢ.833
C − W	2	39	52	Corr.	−	4	45	Corr.			−3.1	G. M. T. $8^h.15$	Corr.	−	6	02	Corr.			+6.8	G. M. T. $8^h.15$
C	8	09	52	* s *h*	48	19	35		3	58	50.9	Corr. −3ˢ.056	* s *h*	23	51	28		21	57	09.8	Corr. +6ˢ.788
c. c.			− 52																		
G. M. T. Jan. 15	8	09	00		′	″		°	′		*H. D.*			′	″			°	′		*H. D.*
R. A. M. S.	19	36	30.2	I. C.	+1	30	Dec. N	19	47.5		o′.04	I. C.	+1	30	Dec.	−9	35.1		+0′.683
Corr.		1	20.3	Dip	−5	22	*p.*	70	12.5	G. M. T. $8^h.15$		Dip	−5	22			+5.6	G. M. T. $8^h.15$
G. S. T.	3	46	50.5	*r*	−	53				Corr. o′.03		*r*	−2	10	Dec.	9	29.5 S.	Corr. +5′.56
				Corr.	−4	45						Corr.	−6	02	*p*	99	29.5	

Line from Jupiter

	°	′	″		
h	48	19	35		
L	39	30	00	sec	0.11259
p	70	12	30	cosec	0.02645
	158	02	05		
s	79	01	03	cos	9.27992
	48	19	35		
$s - h$	30	41	28	sin	9.70792

	h	m	s		
t = −	2	51	44.1	hav	9.12688
R. A. Jupiter	3	58	50.9		
L. S. T.	1	07	06.8		
G. S. T.	3	46	50.5		
Lon.	2	39	43.7 W.		
	39°	55′	55″.5 W. = λ_1		

	°	′	″		
h	48	19	35		
L_2	39	50	00	sec	0.11469
p	70	12	30	cosec	0.02645
	158	22	05		
s	79	11	03	cos	9.27336
	48	19	35		
$s - h$	30	51	28	sin	9.71004

	h	m	s		
t = −	2	51	15.1	hav	9.12454
R. A. Jupiter	3	58	50.9		
L. S. T.	1	07	35.8		
G. S. T.	3	46	50.5		
Lon.	2	39	14.7 W.		
	39°	48′	40″.5 W.		

Lon. Factor $F_1 = 39° 55′ 55″.5 - 39° 48′ 40″.5 = 7′ 15″ = \frac{7′.25}{20′} = 0.363$ and position line runs NE.

Line from Venus

	°	′	″		
h	23	51	28		
L	39	30	00	sec	0.11259
p	99	29	30	cosec	0.00599
	162	50	58		
s	81	25	29	cos	9.17350
	23	51	28		
s − *h*	57	34	01	sin	9.92635
	h	*m*	*s*		
t = +	3	11	57.3	hav	9.21843
R. A. Venus	21	57	09.8		
L. S. T.	1	09	07.1		
G. S. T.	3	46	50.5		
Lon.	2	37	43.4 W.		
	39°	25′	51″ W. = λ_{11}		

	°	′	″		
h	23	51	28		
L_2	39	50	00	sec	0.11469
p	99	29	30	cosec	0.00599
	163	10	58		
s	81	35	29	cos	9.16504
	23	51	28		
s − *h*	57	44	01	sin	9.92715
	h	*m*	*s*		
t = +	3	10	39.1	hav	9.21287
R. A. Venus	21	57	09.8		
L. S. T.	1	07	48.9		
G. S. T.	3	46	50.5		
Lon.	2	39	01.6 W.		
	39°	45′	24″ W.		

Lon. Factor $F_2 = \frac{19'33''}{20'}$ or $\frac{19'.55}{20'}$ = 0.978 and position line runs NW.

Therefore the lines are unlike and $\Delta L(F_1 + F_2) = \lambda_I - \lambda_{II}$.

	°	′	″	
λ_I =	39	55	55.5 W.	$\Delta L(0.363 + 0.978) = 30'.075$
λ_{II} =	39	25	51.0 W.	$\Delta L = \frac{30'.075}{1.341} = 22'.427 = 22'\ 26''$
$\lambda_I - \lambda_{II}$ =		30	04.5	
=			30.075	

$\Delta\lambda_I = \Delta L \times F_1 = 22'.427 \times .363 = 8'.1410 = 8'\ 08''.5$E.
$\Delta\lambda_{II} = \Delta L \times F_2 = 22.427 \times .978 = 21.9336 = 21'\ 56'$ W.

	°	′	″	
λ_I =	39	55	55.5 W.	
$\Delta\lambda_I$		8	08.5 E.	
Lon. =	39	47	47 W.	

	°	′	″	
λ_{II} =	39	25	51.0 W.	
$\Delta\lambda_{II}$		21	56.0 W.	
Lon. =	39	47	47 W.	

	°	′	″	
L_1	39	30	00 N.	
ΔL		22	26 N.	
Lat. =	39	52	26 N.	

EXAMPLE 2: Now work the above example by the tangent method, with the D. R. latitude 40° N. at time of observation.

Time				Jupiter		Venus	
				Corrected Altitude	Corrected R. A. and Dec.	Corrected Altitude	Corrected R. A. and Dec.
	h	*m*	*s*	° ′ ″	*h* *m* *s*	° ′ ″	*h* *m* *s*
G. M. T.	8	09	00	*h* 48 19 35	R. A. 3 38 50.9	h_1 23 51 28	R. A. 21 57 09.8
R. A. M. S	19	36	30.2		Dec. 19° 47′.5 N.		Dec. 9° 29′.5
Corr.		1	20.3		*p* 70 12.5		p_1 99 29.5
G. S. T.	3	46	50.5				

Jupiter

	°	′	″		
h	48	19	35		
L	40	00	00	sec	0.11575
p	70	12	30	cosec	0.02645
	158	32	05		
s	79	16	03	cos	9.27004
h	48	19	35		
s − *h*	30	56	28	sin	9.71110
	h	*m*	*s*		
t = −	2	51	00.2	hav	9.12334
R. A. Jupiter	3	58	50.9		
L. S. T.	1	07	50.7		
G. S. T.	3	46	50.5		
Lon.	2	38	59.8 W.		
or	39°	44′	57″ W. = λ_1		

Venus

	°	′	″		
h	23	51	28		
L	40	00	00	sec	0.11575
p	99	29	30	cosec	0.00599
	163	20	58		
s	81	40	29	cos	9.16075
h	23	51	28		
s − *h*	57	49	01	sin	9.92755
	h	*m*	*s*		
t = +	3	09	59.7	hav	9.21004
R. A. Venus	21	57	09.8		
L. S. T.	1	07	09.5		
G. S. T.	3	46	50.5		
Lon.	2	39	41.0 W.		
or	39°	55′	15″ W. = λ_{11}		

Azimuth Tables

L 40° N.
H. A. $2^h\ 51^m$
d 19° 47′.5 N.

N. 106° 08 E. (S. 73° 52′ E.)
Line runs NE. – SW.
(Tab. 47 Bowd.) $F = 0.37$

Azimuth Tables

L 40° N.
H. A. $3^h\ 10^m$
d 9° 29.5 S.

N. 127° 20′ W. (S. 52° 40′ W.)
Line runs NW. – SE.
(Tab. 47) $F_1 = 1.0$

Therefore the lines are unlike.

$$\left.\begin{array}{rr} \lambda_{II} = & 39^\circ\ 55'\ 15'' \text{ W.} \\ \lambda_{I} = & \underline{39\ 44\ 57} \text{ W.} \\ \lambda_{II} - \lambda_{I} & 10'\ 18'' = 10'.3 \end{array}\right\} L\ 40^\circ \text{ N.}$$

$$\text{but } \lambda_{II} - \lambda_{I} = \Delta L(F + F_1) \therefore 10'.3 = \Delta L(.37 + 1.0).$$

$$\Delta L = \frac{10'.3}{1.37} = 7'.518 = 7'\ 31''$$

Corr. for Lon.

$$\Delta\lambda_{I} = \Delta L \times F = 7'.518 \times 0.37 = 2'.781 = 2'\ 47''$$
$$\Delta\lambda_{II} = \Delta L \times F_1 = 7'.518 \times 1.00 = 7'.518 = 7'\ 31''$$

To find vessel's position:

	°	′	″								
Lat. by D. R.	40	00	00	N.	λ_I	39	44	57	W.	λ_{II}	39	55	15	W.
ΔL_1		7	31	S.	$\Delta\lambda_I$		2	47	W.	$\Delta\lambda_{II}$		7	31	E.
Lat. of position	39	52	29	N.	Lon.	39	47	44	W.	Lon.	39	47	44	W.

(This method is preferable to the chord method.)

EXAMPLE 3. March 21, 1918, in Lat. 34 [illegible], tude of the sun's lower limb was 32° 42′ 30″, I. C. 0′ 0″, height of eye 21 feet, W. T. $9^h\ 00^m\ 00^s$ A.M., C − W $4^h\ 05^m\ 03^s$, chronometer slow $13^m\ 38^s$. Run to noon, compass course, N. 37° W. (323°) Var. 8° W., Dev. 1° E., distance 18 miles, where a meridian altitude was observed and latitude found to be 34° 34′ 35″ N.

Run the Sumner line to noon and find the longitude by observation. (Tangent method.)

Time				Alt.			Corr.			Dec.			Eq. T.		
	h	*m*	*s*	°	′	″		′	″	°	′	*H. D.*		*m*	*s*	*H. D.*
W.	9	00	00	☉	32	42	30	I. C.	+ 0	00	Dec. 0^h	+ 0	01.5	+1′.0	0^h	7	28	−0°.8
C − W	4	05	03	Corr.	+	10	13	S. D.	+16	05	Corr.	+	1.3	$1^h.31$	Corr.		−1	$1^h.31$
Chro.	1	05	03	*h*	32	52	43	Dip	− 4	29	Dec.	0	02.8 N.	+ 1.′3			−7	27	$-1^s.0$
c. c.	+	13	38					*p* & *r*	− 1	23		*p* = 89° 57′ 12″						
G. M. T.	1	18	41					Corr.	+10	13								
Mar. 21	= $1^h.31$																	

	h	*m*	*s*
G. M. T.	1	18	41
Eq. t.	−	7	27
G. A. T.	1	11	14

	°	′	″				
h	32	52	43				
L	34	30	24	sec	0.08404	sec	0.07581
p	89	57	12	cosec	0.00000	sec	0.08404
	157	20	19				
s	78	40	10	cos	9.29330	cos	9.29330
h	32	52	43				
s − *h*	45	47	27	sin	9.85540	(*s* − *p*) cos	9.99152
				hav	9.23274	hav	9.44467

	h	*m*	*s*
L. A. T. =	20	44	39.1
G. A. T.	1	11	14
Lon.	4	26	34.9 W.
or	66°	38′	43″ W.

= S. 63° 42′ E.
or N. 116° 18′ E. = *Z*

The line of position runs NE. – SW.
Tab. 47, *F* = .59

Compass course	Dev.	Var.	True C.	Dist.	N.	W.	D. Lo.
N. 37° W.	1° E.	8° W.	N. 44 W.	18	12.9	12.5	15.2

	°	′	″			°	′	″
Lat. D. R.	34	30	24 N.	Lon. from sight		66	38	43 W.
run = l =		12	54 N.	D =			15	12 W.
Lat. D. R. (noon)	34	43	18 N.	Computed lon. =		66	53	55 W.
Lat. (mer.-alt.)	34	34	35 N.	$\Delta\lambda$			5	09 W.
ΔL		8	43 S.	Lon. by obs. (noon)		66	59	04 W.

$= 8'.72$ S.

$\Delta\lambda = \Delta L \times F = 8'.72 \times .59 = 5'.144 = 5'\ 09''$

Noon position, Lat. 34° 34′ 35″ N. Lon. 66° 59′ 04″ W.

EXAMPLE 4: August 20, 1918, in Lat. 34° 50′ N., Lon. 132° 20′ W., by D. R. the sextant altitude of the sun's lower limb was 37° 00′ 00″, I. C. – 2′ 30″, height of eye 25 feet, W. T. $8^h\ 30^m\ 00^s$ A.M., C – W $9^h\ 07^m\ 37^s$, chronometer fast $18^m\ 17^s$. Work a line of position (Tangent method). From this observation steamed N. 62° E. (true) 28 miles when the sextant observation of the sun's lower limb was observed to be 64° 25′ 00″ bearing southeastward I. C. – 2′ 30″, height of eye 25 feet, W. T. observation $11^h\ 09^m\ 03^s$ A.M., C – W $9^h\ 07^m\ 37^s$, chronometer fast $18^m\ 17^s$. Work the second sight as a $\phi''\ \phi'$ sight and find vessel's position.

Time		Alt.		Alt. Corr.		Dec.		Eq. of T.	
	h m s	° ′ ″		′ ″		° ′	H. D.	m s	H. D.
W.	8 30 00	⊙ 37 00 00	S. D.	+15 50	At 4^h	12 35.0	−0′.8	At 4^h − 3 24.1	− $0^s.6$
C − W	9 07 37	Corr. + 7 16	Dip	− 4 54		1.1	G. M. T. $1^h.32$	Corr. − 0.8	G. M. T. $1^h.32$
C	5 37 37	h 37 07 16	I. C.	− 2 30	d = N.	12 33.9	Corr. − 1′.056	−3 25.3	Corr. $0^s.798$
c. c.	− 18 17		p & r	− 1 10	p =	77 26.1			
G. M. T.	5 19 20		Corr.	+ 7 16					
Aug. 20	= $5^h.32$								

	h m s
G. M. T.	5 19 20
Eq. t.	− 3 23.3
G. A. T.	5 15 56.7

	° ′ ″		
h	37 07 16		
L	34 50 00	sec	0.08575
p	77 26 06	cosec	0.01053
	149 23 22		
s	74 41 41	cos	9.42155
h	37 07 16		
$s - h$	37 34 25	sin	9.78517
		hav	9.30300

	h m s
L. A. T.	20 26 57.0
G. A. T.	5 15 56.7
Lon.	8 48 59.7 W.
or	132° 14′ 55.″5 W.

Azimuth Tables

Lat. 34° 50′ N. } N. 101° 12′ E. or
d 12° 34′ N. } S. 78° 48′ E.
H. A. 8^h 27^m A.M.

Tab. 48 latitude factor of 1st line = f_1 = 4.14

True Course	Dist.	N.	E.	D. Lo.
N. 62° E.	28	13.1	24.7	30.0

	° ′ ″		° ′ ″
Lat. D. R.	34 50 00 N.	Lon. at sight	132 14 56 W.
run	13 06 N.	D =	30 00 E.
Lat. D. R.	35 03 06 N.	Computed Lon.	131 44 56 W.
		=	8^h 46^m $59^s.9$

Now work the ϕ'' ϕ' sight

Time	h	m	s
W.	11	09	03
C − W	9	07	37
C	8	16	40
c. c.	−	18	17
G. M. T.	7	58	23
Eq. t.	−	3	21.8
G. A. T.	7	55	01.2
λ W.	8	46	59.9
t =	0	51	58.7
t =	12°	59′	41″

Alt.	°	′	″
☉	64	25	00
Corr.	+	8	02
h	64	33	02

Alt. corr.	′	″
S. D.	+15	50
Dip	− 4	54
I. C.	− 2	30
p & *r*	−	24
Corr.	+ 8	02

Dec.	°	′	H. D.
At 6^h	+12	33.4	−0′.8
Corr.	−	1.6	1^h.97
N.	12	31.8	−1.57

Eq. of T.	m	s	H. D.
At 6^h	−3	23	−0^s. 6
Corr.	−	1.2	1^h.97
	−3	21.8	−1^s.18

h	64°	33′	02″
t	12	59	41
d	12	31	48 N.
ϕ''	12	50	53 N.
ϕ'	22	15	20 N.
Lat.	35	06	13 N.

sec 0.01127 ; tan 9.34683 ; tan 9.35810 ; sin 9.95567 ; cosec 0.66363 ; sin 9.34708 ; cos 9.96638

Az. Tables
L 35° N.
d 12°.32 N.
H. A. 0^h 52^m
} N. 149° 17′ E. S. 30° 43 E.

Tab. 48, f_2 = .49

With Lon. 131° 44′ 56″ W.

	°	′	″
L′	35	06	13
L	35	03	06

on same meridian $L' - L$ = 3 07 = 3′.117

To find the corr. in latitude

but $L^1 - L = \Delta\lambda(f_1 - f_2)$ ∴ 3′.117 = Δλ(4.14 − .49)]

$$\Delta\lambda = \frac{3.117}{3.65} = .853 = 51''$$

$\Delta L = \Delta\lambda \times f_1 = .853 \times 4.14 = 3'.536 = 3'\ 32''$
$\Delta L^1 = \Delta\lambda \times f_2 = .853 \times .49 = 0'.418 = 0'\ 25''$

To find position

	°	′	″
L =	35	03	06 N.
Δ*L* =		3	32 N.
Lat. =	35	06	38 N.

	°	′	″
L^1 =	35	06	13 N.
Δ*L*^{1} =		25	N.
Lat. =	35	06	38 N.

	°	′	″
λ	131	44	56 W.
Δλ		51	E.
Lon.	131	44	05 W.

The lines are alike ΔL and ΔL_1 are both applied the same way to the northward to give same latitude of vessel's position.

The first line runs northeastward and ΔL being to the northward $\Delta\lambda$ is to eastward.

278. The correction for the run of the vessel between two observations may be determined as follows:

To reduce an observed altitude for a change of position of the observer.

Solution. Let

Z (Fig. 54) be the zenith of the place of observation;

$h = 90° - Z\,m$, the observed altitude;

Z′, the zenith of the new position;

$h' = 90° - Z'\,m$, the altitude reduced to the new position, Z'.

$d = Z\,Z'$, the distance of the two places, here referred to the the celestial sphere;

$C = P\,Z\,Z'$, the course;

$Z = P\,Z\,m$, the azimuth of m;

$Z - C = m\,Z\,Z'$, the difference of the course and azimuth

Z Z′, being small, may be regarded as a right line,

Z Z′ O as a plane right triangle, and

O m, without material error, as equal to Z′ m; so that we shall

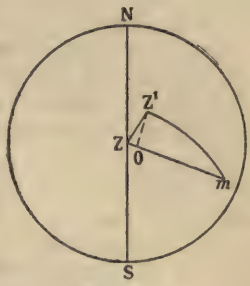

FIGURE 54

have
$$Z\,O = Z\,Z'\ \cos\ Z'\,Z\,m$$
$$Z'\,m = Z\,m - Z\,O$$

or putting
$$\left.\begin{aligned} \Delta h &= Z\,O, \\ \Delta h &= d\cos(C - Z) \\ h' &= h + \Delta h \end{aligned}\right\} \qquad (144)$$

$\Delta h = d\cos(C - Z)$ is, then, the reduction of the observed altitude to the new position of the observer: it is *additive* when $C - Z < 90°$ numerically; *subtractive* when $C - Z > 90°$. It is smaller, and can, therefore, be more accurately computed the nearer $C - Z$ approaches 90°. It is, therefore, better to reduce that altitude for which the difference of the course and azimuth is nearest 90°.

If the second is the one reduced, then C is the opposite of the course.

In practice Z Z′ does not usually exceed 30′, so that although an arc of a great circle of the celestial sphere, it may be regarded as representing the distance, d, of the two places on the

earth; or, at sea, the distance run. The azimuth, or bearing, of the body can be observed with a compass, or be computed to the nearest degree, or half-degree, from the altitude.

The assumption, $Z'm = Om$, is more nearly correct, the greater $Z'm$ or Zm, that is, the smaller the altitude. If we treat $ZZ'm$ as a spherical triangle, $d = ZZ'$ being expressed in minutes and still very small, we shall find

$$\Delta h = d \cos (C - Z) - \tfrac{1}{2} d^2 \sin 1' \tan h \sin^2 (C - Z); \qquad (145)$$

but the last term is inconsiderable unless d and h are both large. For instance, if $d = 30'$, it will not exceed $1'$ unless $h > 82°$.

Should the course be directly or obliquely toward the body the correction is plus; if it is directly or obliquely away from the body the correction is minus; if it is at right angles to the bearing the correction is 0.

EXAMPLE 1: May 19, 1918, in Lat. 40° N., Lon. 60° W., by D. R. about $8^h\ 00^m\ 00^s$ A.M. the sextant altitude of the sun's lower limb was 35° 21′ 40″, I. C. − 2′ 30″, height of eye 20 feet, after running N. 40° E. (true) 40 miles, observed a second altitude of the sun's lower limb 57° 07′ 30″. Find what the first altitude would have been had it been observed at the same time at the second place.

Course N. 40 E.
Az (z) N. 93 E.

					°	′	″
$(C - Z)$	= 53.....	cos	9.77946	at 1st place ☉	35	21	40
d	= 40′.....	log	1.60206	Δh	+	24	04
Δh	= 24′.072 =	log	1.38152	at 2d place	35	45	44

EXAMPLE 2: July 1, 1918, the observed sextant altitude of the sun's lower limb was 30° 54′ 26″, azimuth N. 157° 30′ E., then ran 35 miles N. 11° 15′ E. Find the correction for run.

Azimuth N. 157° 30′ E.
Course N. 11 15 E.

				(or Tab. 2, Bowd.)		
$C - Z$ =	146 15	cos	−9.91985	course	Dist.	D. L.
d =	35	log	1.54407	146° 15′	35	29.1
Δh = −29′.1 =		log	1.46392			

or 29′ 06″ subtracted from altitude

279. By (144) or (145) we may reduce one of the two altitudes for the change of the vessel's position in the interval. But instead of this we may put down the line of position for

each observation, and afterwards move one of them to a parallel position determined by the course and distance steamed in the interval. Thus, in Fig. 55, let

A B be the line of position for the first observation, and

A *a* represent in direction and length the course and distance steamed in the interval; then *a b*, drawn parallel to A B, is the line of position which would have been found had the first altitude been observed at the place of the second.

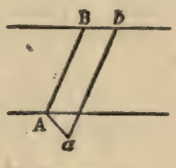

FIGURE 55

If the second observation is to be reduced to the place of the first, then A *a* in direction must be the opposite of the course.

The perpendicular distance of A B and *a b* is the reduction of the altitude for the change of position: for that distance is A *a* × cos (B A *a* − 90°).

CHAPTER XIV

THE NEW NAVIGATION

280. This is a method devised by the late Admiral Marcq St. Hilaire, a distinguished officer of the French Navy, as long ago as 1875, for fixing the vessel's position by two combined altitudes of the sun, or other celestial bodies, separated by a suitable difference of bearing. The system adopted is to compute the altitude, or zenith distance of the body used, and compare this with the observed altitude, then find the bearing of

FIGURE 56

the body by computation, or azimuth tables, and on this bearing mark off the *intercept* or *altitude difference* between the observed and computed zenith distance, toward or away from the body, as the case may be, and from this point plot a position-line at right-angles to the bearing of the body. The intersection of the two lines is the position of the vessel.

This is but another method of determining a line of position, or Sumner line, and among intelligent navigators both of the navy and merchant marine, this method has almost entirely supplanted the old time sight for longitude, and the chord and tangent method for obtaining the Sumner lines, because it has the great advantage of the same simple method for the solution of all sights without limitation as to altitude, azimuth, or hour-angle, and the navigator need concern himself little as to whether the sight should be taken near the prime vertical, or as a ϕ'' ϕ' sight, or as a circummeridian.

281. Suppose a navigator at a certain G. M. T. observes the altitude of the sun or any celestial body, and it is found to be 30° the zenith distance is therefore 60°. In Fig. 56 let S be the geographical position of the body; then this one observation shows that the observer is at a distance of 60° or 3600 nautical miles, that is to say he is somewhere on the circumference of the circle of altitude $D\,A\,M$, the center of which is S and radius $S\,A$ (3600 nautical miles), and this circle of altitude passes through his zenith. He may be presumed to be on that portion of the circumference which passes in the locality of his dead-reckoning position, and intersecting the circle in the direction of the azimuth of the body. Let B be the position by dead reckoning. By spherical trigonometry, using the cosine-haversine formula (Art. 161),

$$\begin{aligned} &\text{hav } \theta = \cos \text{L} \cos d \text{ hav } t; \\ &\text{hav } z = \text{hav } (L \mp d) + \text{hav } \theta \end{aligned} \qquad (73)$$

the zenith distance of the sun or star at B can be easily computed for the instant of observation, for we know the D. R. latitude, the declination of the body and the hour-angle on the meridian of B.

In Fig. 56 suppose the zenith distance at B is computed for the instant of observation and found to be 60° 10′, and that the true bearing of the body from B is 218°, at the same instant; then $S\,B$ = 3610 nautical miles, and $A\,B$ = 10 miles; azimuth 218°, that is $A\,B$ is the difference between the true zenith distance found by observation with a sextant at the vessel, and the computed zenith distance for the D. R. position B, or it is the difference between the true altitude and the computed altitude.

If we neglect the spheroidal form of the earth, the azimuth of a celestial body is the same as the angle at the place between the meridian of the place and the great circle which joins

the place to the geographical position of the celestial body, or in other words the true bearing of the geographical position of a celestial body is the same as the azimuth of the celestial body.

A part of the arc $L\,C\,B$, at B, and the arc $B\,A$, will be shown in Fig. 57 as it is drawn on a Mercator chart.

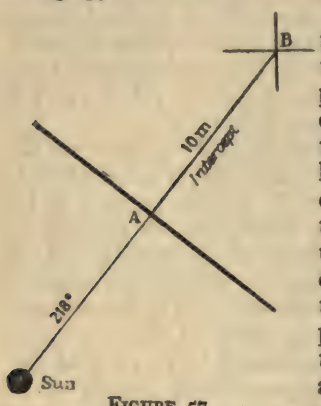

FIGURE 57

282. Since the arcs are extremely short in comparison to their radii, they will be for all practical purposes straight lines coinciding with rhumb-lines, so that to lay off from B a straight line $B\,A$, known as the *intercept*, or *altitude difference*, representing ten miles (or the difference in minutes of arc between the true and computed zenith distance) *towards* the celestial body if the computed zenith distance be greater than the true zenith distance, and *away* from the body, if the computed zenith distance be less than the true zenith distance. This is evident from Fig. 56, because to get to the true circle of altitude or position circle $D\,A\,M$, from the computed circle of position, $L\,C\,B$, go toward the celestial body, that is in the direction of its azimuth. It will be seen that the line of position is always perpendicular to the azimuth line from the celestial body when drawn on the chart.

From the above remarks, it is evident that the problem of the new navigation, when determining a line of position, is always to find how far in miles the line of position is located from the position by dead-reckoning, and in what direction it lies. It is well to consider also that one observation will not determine the position of a vessel, but only locate it somewhere on a line of position, which is as valuable and as good information as the mere knowledge of a latitude or longitude determined at sea, which in nearly all cases is in error and not the exact location of the vessel.

EXAMPLE 1: November 20, 1918, in Lat. 19° 15′ N., Lon. 84° 06′ W., by D. R. the sextant altitude of the sun's lower limb was 40° 43′ 30″, I. C. + 0′ 30″, height of eye, 39 feet; W. T. $9^h\ 55^m\ 55^s$ A.M., C − W $5^h\ 19^m\ 49^s$, chronometer slow $3^m\ 02^s$. Find a position point, and a line of position by the method of Marcq St. Hilaire.

Time	h	m	s
W.	9	55	55
C – W	5	19	49
C.	3	15	44
e. c.	+	3	02
G. M. T., Nov. 20	3	18	46
Eq. t.	+	14	24
G. A. T.	3	33	10
Lon. W.	5	36	24
L. A. T.	21	56	46
t = E.	2	03	14
or	30°	48′	30″ E.

Alt.	°	′	″
☉	40	43	30
Corr.	+	9	35
h	40	53	05
z =	49	06	55

Alt. Corr.	′	″
S. D.	+16	13
I. C.	+ 0	30
Dip	– 6	07
p & *r*	– 1	01
Corr.	+ 9	35

Dec.	°	′	
At 2^h	–19	35.9	
Corr.		+.8	
	S 19	36.7	
H. D.		+0′.9	
G. M. T.	$1^h.31$		
Corr.		+0′.78	

Eq. T.	m	s
At 2^h	+14	24.8
Corr.		–.8
	+14	24

	°	′	″				*Azimuth*	
t	30	48	30	hav		8.84854	sin	9.70942
L	19	15	00 N.	cos		9.97501		
d	19	36	42 S.	cos		9.97405	cos	9.97405
				hav		8.79760		
				nat. hav		.06275		
(*L* — *d*)	38	51	42	nat. hav		.11067		
comp. *z* =	49	13	10	nat. hav		.17342	cosec	0.12078
true *z* =	49	06	55				sin	9.80425 = *Z* = S. 39° 35′ E.
Intercept =		6	15 towards sun					
=		6′.2						

By traverse tables

Course	Dist.	D. L.	Dep.	D. Lo.
S. 39°.6 E.	6.2	4.76	3.95	4.18

	°	′	″				
Lat. D. R.	19	15	00 N.		Lon.	84	06 00 W.
l		4	45 S.		*D* =		4 11 E.
Lat. of point *C*	19	10	15 N.		Lon. of point C	84	01 49 W.

This example is plotted on the chart, as in Fig. 58.

The Hydrographic Office issues publication No. 200, Altitude Azimuth and Line of Position Tables (Aquinos Method), which

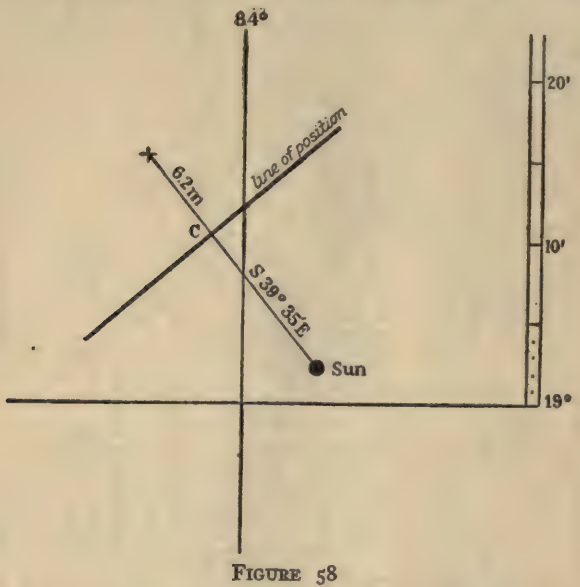

FIGURE 58

permits the above example to be solved readily by inspection, and the next example will be worked by these spherical traverse tables.

EXAMPLE 2:

Data from preceding example. (H. O. Pub. 200) (Aquinos Method)

Time				True Alt.			Dec.		
	h	*m*	*s*	°	′	″	°	′	″
G. A. T., Nov. 20	3	33	10	40	53	05	S 19	36	42
or									
Arc	53°	17′	30″W.	Arc	53°	17′	30″W.		
$\lambda_{D.R.}$	84	06	00 W.	t_A	30	58	06 W.		
$t_{D.R.}$	30	48	30 W.	λ_A	84	15	36 W.	} Pos. A.	
a	29	00	00	L_A	19	26	08 N.		
h_0	40	53	05	b	22	33	52		
h_C	40	32	00	C	42	00	00		
Intercept	21	05	towards sun	Z	39	38	00		

Course	Dist.	D.Lo.	Dep.	D. Lo.
S. 39° 38′ E.	21.1	16.25	13.46	14.26

	°	′	″			°	′	″
L_A	19	26	08 N.	λ_A		84	15	36 W.
	16	15	S.	D			14	16 E.
Lat. of point D,	19	09	53 N.	Lon. of point D,		84	01	20 W.

The line of position determined by inspection (spherical traverse) and also by the cosine-haversine formula is plotted as in Fig. 59.

It is seen that the assumed Position A differs greatly in Lat. and Lon. from the position by D. R. used in the cos-hav

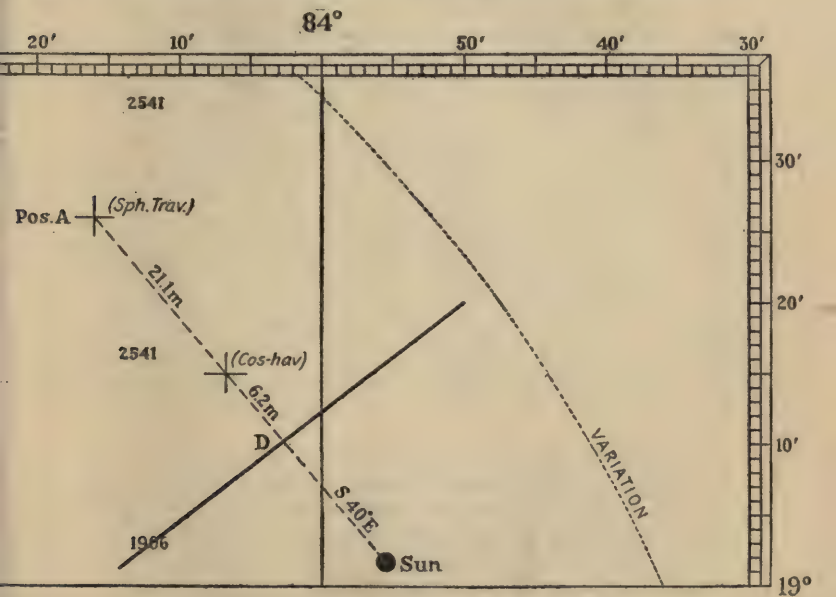

FIGURE 59

method, but the interceps also differ, making the line of position exactly the same in either case, so that it makes little difference where the origin is chosen. Using the cosine-haversine method it is better to use a position near the position by D. R.

283. *Position from two or more observations.* — The position of a vessel is found from the intersection of two or more position

lines. It is important to obtain the position from two or more simultaneous or nearly simultaneous observations, in order that the accuracy of the position may not be dependent on that of the run of the vessel.

The best time for taking observations is at morning or evening twilight, when the stars and planets are just visible and the horizon is usually very clearly defined; it should, therefore, be made a rule that, whenever the vessel's position is not accurately known, observations of two or more stars or planets should be taken at morning and evening twilight. When selecting celestial bodies to observe, it should be remembered that the accuracy of the position depends largely on the angle at which the position lines cut one another; this angle is the same as the difference of the azimuths of the bodies, so that bodies should be selected the difference of whose azimuths is as near 90° or a right angle as possible, and never less than 30°.

When the intercepts have been found for two observations, the position of the vessel, which is at the intersection of the two corresponding position lines, may be found by plotting or by computation. When the chart is on a sufficiently large scale the method by plotting on the chart has the advantage that the position of the ship with regard to the land or soundings is immediately known. When the vessel is off soundings the chart usually is of a small scale, hence difficult to plot position lines. The Hydrographic Office issues a series of plotting sheets on a sufficiently large scale, covering navigable parts of the ocean, therefore very useful for plotting lines of position.

EXAMPLE 3: May 1, 1918, in Lat. 49° 55′ N., Lon. 7° 15′ W., by D. R. observed simultaneous altitudes of star Procyon (α Can. Min.), bearing southwestward and star Capella (α Aurigæ) bearing northwestward, sextant altitude of Procyon, 35° 40′ 00″, W. T. $7^h\ 28^m\ 04^s$, P.M., $C - W\ 0^h\ 39^m\ 33^s$, chronometer fast $10^m\ 00^s.7$, sextant altitude of Capella 42° 34′ 00″, W. T. $7^h\ 29^m\ 51^s$ P.M., I. C. + 1′ 30″, height of eye 40 feet. Find position lines and intersection. (Marc St. Hilaire Method.)

(Marc St. Hilaire Method)

Time	h	m	s
W.	7	28	04
C – W	0	39	33
C.	8	07	37
c. c.		– 10	00.7
G. M. T., May 1	7	57	36.3
Lon. W.	0	29	00.0
L. M. T.	7	28	36.3
R. A. M. S.	2	35	43.5
L. S. T.	10	04	19.8
R. A. *	7	35	02.2
t	2	29	17.6
t = 37° 19′ 24″			

Alt.	°	′	″
*'s h_o	35	40	00
Corr.	–	6	03
h	35	33	57
z	54	26	03

Alt. Corr.	′	″
I. C.	+1	30
Dip	–6	12
r	–1	21
Corr.	–6	03

* Procyon R. A. & Dec.	h	m	s
R. A.	7	35	02.2
Dec.	N.5°	26′	.0

R. A. M. S.	h	m	s
	2	34	25.0
+	1		18.5
	2	35	43.5

	°	′	″
t	37	19	24
L	49	55	00 N.
d	5	26	00 N.
(*L* — *d*)	44	29	00
Comp. *z* =	54	23	40
true *z* =	54	26	03

			Azimuth	
hav	9.01024		sin	9.78270
cos	9.80882			
cos	9.99804		cos	9.99804
hav	8.81710			
nat. hav	.06563			
nat. hav	.14327			
nat. hav	.20890		cosec	0.08989
			sin	9.87063 = *Z*
				= S. 47° 56′ W.

Intercept 2 23 away from Procyon
= 2′.4

Time				Alt.			Alt. Corr.			* Capella R. A. and Dec.			R. A. M. S.		
	h	*m*	*s*	°	′	″		′	″		*h*	*m*	*s*	*h*	*m*	*s*
W	7	29	51	*'s h_0	42	34	00	I. C.	+1	30	R. A.	5	10	39.1	2	34	25.0
C – W	0	39	33	Corr.	–	5	45	Dip	−6	12	Dec.	N.45°	55′.1		+ 1	18.8
C	8	09	24	*h*	42	28	15	*r*	−1	03					2	35	43.8
c. c.		−10	00.7	*z*	47	31	45	Corr.	−5	45							
G. M. T., May 1	7	59	23.3														
Lon. W.	0	29	00.0														
L. M. T.	7	30	23.3														
R. A. M. S.	2	35	43.8														
L. S. T.	10	06	07.1														
R. A. *	5	10	39.1														
t =	4	55	28														
t =	73°	52′	00″														

	°	′	″			*Azimuth*
t =	73	52	00	hav	9.55758	sin 9.98255
L =	49	55	00 N.	cos	9.80882	
d	45	55	06 N.	cos	9.84241	cos 9.84241
				hav	9.20881	
				nat. hav	.16174	
(*L* — *d*)	3	59	54	nat. hav	.00122	
Comp. *z*	47	37	00	nat. hav	.16296	cosec 0.13156
True *z*	47	31	45			sin 9.95652 = *Z*.
Intercept		5	15 towards Capella			= N. 64° 47′ W.
		= 5′.2				

The lines are plotted as in Fig. 60.

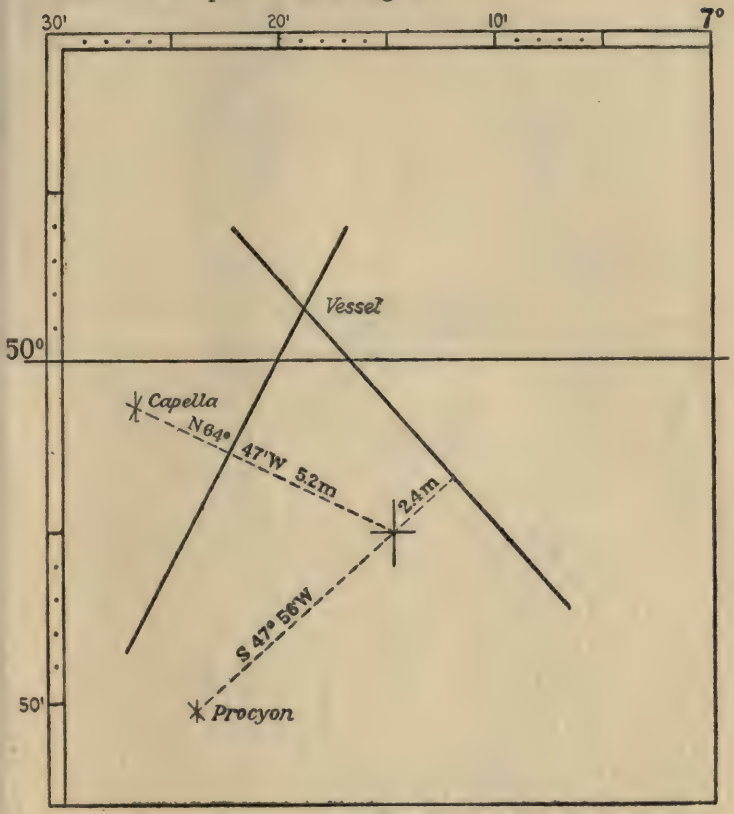

FIGURE 60

EXAMPLE 4: May 14, 1918, in Lat. 38° 44′ N., Lon. 72° 46′ W., by D. R., the sextant altitude of the sun's lower limb bearing northeastward, was 42° 21′ 30″, W. T. $8^h\ 38^m\ 00^s$ A.M., C − W $4^h\ 51^m\ 25^s$, chronometer fast $2^m\ 10^s.5$, I. C. + 1′ 00″, height of eye 30 feet. From this position steamed N. 52° E. (true) 22 miles when the sextant altitude of the sun's lower limb bearing southeastward was 64° 04′ 30″, W. T. $10^h\ 49^m\ 30^s$ A.M., C − W $4^h\ 47^m\ 25^s$, chronometer fast $2^m\ 10^s.5$, I. C. + 1′ 00″, height of eye 30 feet. Find the position of the vessel at the second observation, using St. Hilaire method.

First Line

Time				Alt.			Alt. Corr.		Dec.			Eq. of T.	
	h	*m*	*s*	°	′	″	′	″	°	′	*H. D.*	*m*	*s*
W.	8	38	00	☉ 42	21	30	S. D. +15	51	At 0^h +18	29.8	0′.6	+3	48.5
C − W	4	51	25	Corr. +	10	32	I. C. + 1	00	Corr. +	9	1^h.45		
C.	1	29	25	*h* 42	32	02	Dip − 5	22	Dec. N. 18	30.7	Corr. + 0.9		
c. c.	−	2	10.5	*z* 47	27	58	*p* & *r* −	57					
G. M. T., May 14	1	27	14.5				Corr. +10	32					
Eq. t.	+	3	48.5										
G. A. T.	1	31	03										
Lon. W.	4	51	04										
L. A. T.	8	39	59										
t = −3		20	01										

t	3^h	20^m	01^s	hav	9.25196
	°	′	″		
L	38	44	00 N.	cos	9.89213
d	18	30	42 N.	cos	9.97693
				hav	9.12102
				nat. hav	.13214
(*L* — *d*)	20	13	18	nat. hav	.03082
Comp. *z* =	47	37	00	nat. hav	.16296
True *z* =	47	27	58		
Intercept		9	02 towards sun		

Azimuth from tables
S. 80° E.
or N. 100° E.

Course	Dist.	D. L.	Dep.	D.Lo.
54°	22	12.9	17.8	22.9

	°	′	″				
Lat. D. R.	38	44	00 N.	Lon. D. R.	72	46	00 W.
l		12	54 N.	*D*		22	54 E.
Lat. new position pt.	38	56	54 N.	Lon. new pos. pt.	72	23	06 W.

Second Line

Time				Alt.			Alt. Corr.			Dec.				Eq. T.	
	h	*m*	*s*	°	′	″		′	″		°	′	*H. D.*	*m*	*s*
W.	10	49	30	☉	64	04	30	S. D.	+15	51	At 2^h	+18	31.0	+3	48.5
C – W	4	47	25	Corr.	+	11	05	I. C.	+ 1	00		+	.9		
C.	3	36	55	*h*	64	15	35	Dip	− 5	22	Dec. N.	18	31.9	+0.9	
c. c.	−	2	10.5	*z*	25	44	25	*p* & *r*	−	24					
G. M. T., May 14	3	34	44.5					Corr.	+11	05					
Eq. t.	+	3	48.5												
G. A. T.	3	38	33.0												
Lon. W.	4	49	32.4												
L. A. T.	10	49	00.6												
t = −	1	10	59.4												

t	1^h	10^m	$59^s.4$	hav	8.37649
	°	′			
L	38	56	54 N.	cos	9.89082
d	18	31	54 N.	cos	9.97687
				hav	8.24418
				nat. hav	.01755
(*L* — *d*)	20	25	00	nat. hav	.03141
Comp. *z* =	25	34	00		.04896
true *z* =	25	44	25		
Intercept		10	25 away from sun		

Azimuth from tables
N. 138° E.

This example is plotted on the chart, as shown in Fig. 61 (from a section of a H. O. Plotting sheet).

284. The position line for the noon latitude obtained from a meridian altitude is drawn perpendicular to the meridian through the determined latitude.

The position line for a Polaris sight, or a circummeridian is drawn perpendicular to the azimuth of the body at the instant of observation, through the determined latitude.

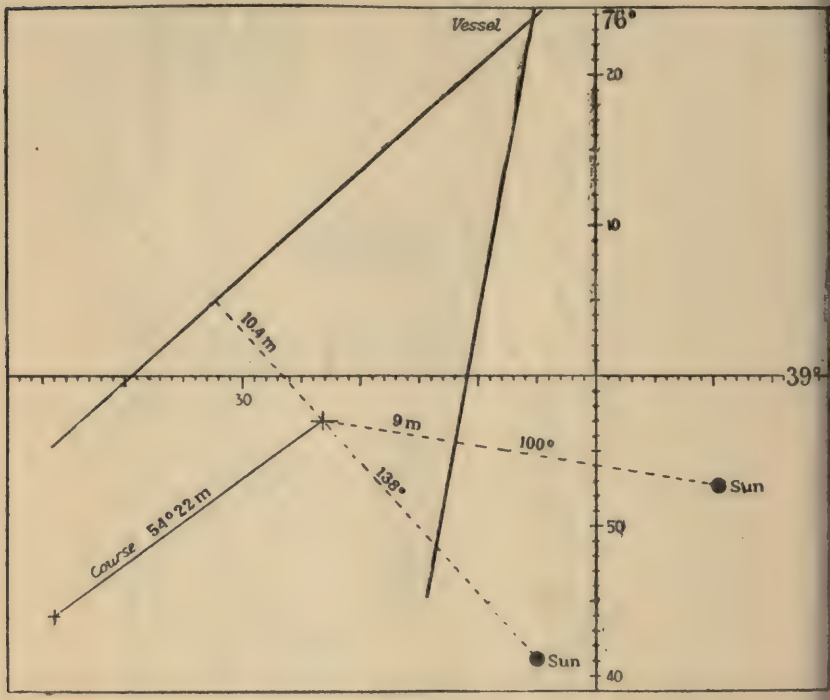

FIGURE 61

This represents all the cases of the new navigation and all that is required for the navigation of a vessel upon the oceans.*

* The Hydrographic Office has issued a new publication of Altitude and Azimuth Tables computed by the Cosine-haversine method (Formulaes 73 and 76) in which the tables are entered with the given Latitude, Declination, and Hour Angle (given to every 10 minutes of time), and by simple inspection the combined altitude and the azimuth of the celestial body is given thus eliminating all computation other than simple interpolation. These tables will give the altitude and azimuth as determined in the above problems without computation on the part of the student or navigator, and will no doubt prove of more practical value than H. O. Pub. No. 200 mentioned above, because of the complex method of the latter publication.

CHAPTER XV

CELESTIAL BODIES AND THEIR IDENTIFICATION

285. *Astronomical observations.* — The position of the vessel should be frequently determined by observation, not only as a check on the accurate steering of the course and on the estimated speed, but also to guard against a wrong estimation of currents, tidal streams, etc. When it is impossible to determine the vessel's position by observations of fixed objects on shore, as, for instance, when there is a haze over the land, or when the vessel is in mid ocean, the position must be found by observation of the celestial bodies. It is fortunate that the navigator need not depend on observations of the sun alone, but may turn to the planets or fixed stars, and the use of stars over solar observations is of great benefit to the navigator.

286. The two great advantages of stellar over solar observations are (first) that by the stars the latitude and longitude can be obtained simultaneously, as in Example 1, Art 274, instead of having an interval of three or four hours between the observations as is often the case when the sun is used, and (second) that uncertain errors in altitude resulting from personal equation, arc errors of sextant, or exceptional refraction may be practically eliminated by a proper choice of stars. If meridian altitudes can be obtained about the same time to the north and south of the observer, it will be apparent to any one that the errors would be eliminated by taking the mean result of the two observations for the true latitude; and in the same way the true longitude would be found by the mean result of observations taken nearly east and west of the meridian. Of course it may happen that stars are not to be found north, south, east, and west of the observer during the short time that the horizon is good for observation. The following is one plan suggested in using stars.

Take three stars, and of these choose two on the same side of the meridian northward and southward of the observer for a good latitude. Then choose another star on the other side of the meridian, as near as possible the same distance from the meridian; calculate the longitude with the latitude found from the previous observations, and the true longitude will be obtained by taking the mean between the eastern and western stars worked with the correct latitude.

Avoid star sights when the horizon is uncertain. It is necessary now to give a brief explanation of the stars useful to the navigator.

287. *The stars.* — The stars are bodies comparable in size and physical conditions with the sun, shining by their own light as does the sun, and emitting a radiance which cannot be distinguished from sunlight. For the most part they keep their relative configurations unchanged no matter how much they alter their positions in the heavens from hour to hour. Certain of the heavenly bodies, and among them the sun and moon, continually change their places, moving eastward among the stars. These are the wanderers or planets. They shine with a steady light, while the stars have a twinkling light. There are only four planets useful to the navigator, Venus, Jupiter, Mars, and Saturn, the first two being used the most in practical navigation.

288. *The constellations.* — In ancient times the stars were grouped into constellations, partly as a matter of convenient reference, but generally because of superstition. The names of the constellations are given in a Latinized form, such as Leo, Taurus, Argus, etc.

289. *Designation of bright stars.* — The stars in a particular constellation are designated by letters of the Greek alphabet, assigned usually in order of brightness. Thus the brightest star in the constellation Taurus is α Tauri, the next brightest star is β Tauri, and so on. Some of the bright stars have names of their own, the majority of the names being of Greek or Latin origin, as, for instance, Arcturus (α Boötis), and Procyon (α Canis Minoris); some, however, have Arabic names, as, for instance, Aldebaran (α Tauri).

290. *Magnitudes of stars.* — The term "magnitude," as applied to a star, refers to its brightness. The magnitudes of the stars have been determined on the assumption that the magnitude of a particular star, called Polaris, is 2.1. Magnitudes are assigned to stars according to their brightness,

the brighter the star the smaller being the number assigned to its magnitude; thus Arcturus, the brightest star in the northern heavens, has a magnitude of 0.2.

Those stars whose magnitudes are less than 2 are called stars of the 1st magnitude, those whose magnitudes are 2 and above but less than 3 are called stars of the 2d magnitude, and so on.

Those stars which can only just be seen by the naked eye on a clear night are stars of the 6th magnitude.

The nautical almanac gives the magnitude of the stars, also the time when they transit the meridian.

291. *Identification of stars.* — Most navigational stars can be identified by referring them to principal stars in such well-known constellations as Ursa Major or the Dipper, Orion, and the Southern Cross. Polaris (α Urs. Min.), a line from β Ursa Majoris through Dubhe (α Urs. Maj.), points to the Pole star. α and β Ursa Majoris are called "the pointers."

Arcturus (α *Boötis*) *and Spica* (α *Virginis*). — Arcturus is a bright star and can be found by continuing the curve in the handle of the dipper. This curve continued still farther passes through Spica, Arcturus being midway. Arcturus, Spica, and Regulus, or Arcturus, Spica, and Antares, form bold triangles, the latter triangle right angled at Spica.

Denebola (β *Leonis*). — Denebola forms with Arcturus and Spica an equilateral triangle.

Capella (α *Aurigæ*). — Capella lies on a line drawn from Polaris perpendicular to the line joining the pointers away from the small dipper and at a distance of 45°. Near Capella to the southward is a small isosceles triangle called "the kids."

Vega (α *Lyræ*). — Vega is found by extending the line joining Capella to the Pole Star to about an equal distance on the opposite side of the pole. Near Vega there are five other stars forming a W.

The constellation of Orion is always very conspicuous, and, owing to the large number of bright stars around it, is very useful for pointing out the other stars. Four of the stars in this constellation are at the corners of a quadrilateral, three of them being stars of the 1st magnitude, as Betelguese (α Orionis), Rigel (β Orionis), and Bellatrix (γ Orionis), the other a star of the 2d magnitude (κ Orionis). Three stars are in the middle in a straight line, close together, forming what is called "Orion's belt."

Sirius (α *Canis Majoris*), *and Aldebaran* (α *Tauri*). —

Orion's belt points in one direction approximately towards Sirius, and in the other approximately towards Aldebaran, and that these stars are nearly equidistant from the belt. Sirius has a magnitude of − 1.6, and is the brightest star in the heavens. Aldebaran may be distinguished by its reddish tint.

Canopus (*α Argus*). — Canopus is about 36° S. 5° W. from Sirius, and has a magnitude of − 0.9, being next to Sirius in brightness.

Pollux (*β Geminorum*) *and Castor* (*α Geminorum*). — A line from Rigel through the middle star of Orion's belt points to Castor, and Pollux will be found not far away.

Procyon (*α Canis Minoris*). — Procyon forms, with Sirius and Betelguese, an equilateral triangle.

Regulus (*α Leonis*). — A line from Bellatrix through Betelguese points towards Regulus, which is about 60° from Betelguese.

292. *The constellations of Pegasus and Cassiopeia.* — A line from the pointers through Polaris produced beyond Cassiopeia leads to Scheat (*β* Pegasi), then to Markhab (*α* Pegasi), two stars in a figure called "the square of Pegasus, the other two stars being Algenib (*γ* Pegasi) and Alpheratz (*α* Androm.). Nearly the same distance from Polaris as the Dipper, but on the opposite side, is Cassiopeia's chair, whose stars appear as an M or a W, according to the position of the constellation.

Markab (*α Pegasi*). — This star is the southwestern one of the square of Pegasus, and the two western stars of the square point to Polaris.

Fomalhaut (*α Piscis Australis*). — The line joining the two western stars of Pegasus, produced away from Polaris, passes through Fomalhaut.

Altair (*α Aquilæ*). — Altair lies between two less brilliant stars in a line with Vega; a line from Capella through *γ* and *β* Cassiopeiæ points to Altair.

Antares (*α Scorpii*). — Antares is a reddish star, and is at the center of a small bow which points directly at another bow.

Achernar (*α Eridani*). — Achernar lies on a line joining Canopus and Fomalhaut, and is midway between them.

293. *The Southern Cross.* — In the southern hemisphere the most conspicuous constellation is the Southern Cross. It is outlined by four bright stars. *α*, *β*, and *γ* Crucis are all stars of the 1st magnitude. As a line through *α* and *β* Centauri points to the Cross, those stars are called also the pointers. *β* Centauri is nearer the Cross than *α* Centauri.

EXAMPLE 1: November 17, 1918, in Lat. 40° N., Lon. 140° W., by D. R., weather cloudy, a bright star was observed through a break in the clouds, on the meridian to the southward, star's sextant altitude 33° 30′ 00″, I. C. + 1′ 30″, height of eye 30 feet, W. T. $3^h\ 00^m\ 00^s$ A.M., C − W $9^h\ 23^m\ 30^s$, chronometer fast $3^m\ 30^s$. Find what star was observed.

Time	h	m	s
W.	3	00	00
C − W	9	23	30
C	0	23	30
c. c.	−	3	30
G. M. T., Nov. 17.	0	20	00
R. A. M. S.	15	42	56
Corr. for G. M. T.		0	03.2
G. S. T.	16	02	59.2
Lon. W.	9	20	00.0
L. S. T. or *'s R. A.	6	42	59.2

Alt.	°	′	″	
*'s h_0	33	30	00	
Corr.	−	5	19	
$*h$	33	24	41	S.
$*z$	56	35	19	N.
Lat.	40	00	00	N.
$*d$	16	35	19	S.

Alt. Corr.	′	″
I. C.	+1	30
Dip	−5	22
r	−1	27
Corr.	−5	19

An inspection of the almanac shows the star to be Sirius, having a R. A. of $6^h\ 41^m\ 36^s$. Dec. 16° 36′.3. S.

EXAMPLE 2: February 26, 1918, in Lat. 35° 20′ N., Lon. 60° W. by D. R., at $6^h\ 20^m$ P.M. Weather overcast and cloudy. Observed with sextant an altitude of an unknown star through a break in the clouds to be 31° 30′ (true), bearing 285° (true). What is the name of the star?

L	35° 20′	log sin	9.762	log cos	9.912		
h	31° 30′	log sin	9.718	log cos	9.931	log cos	9.931
Z	285° 00′			log cos	9.413	log sin	9.985
A	0.302	log	9.480				
B	0.180			log	9.256		
A + B = 0.302 + 0.180 = 0.482 = nat sin d ∴ d = 28° 49′						log sec	10.057
t = H. A. = 70° = $4^h\ 40^m$						log sin	9.973

Converting the hour-angle into right ascension, we have

	h	m
L. M. T.	6	20
R. A. M. S.	22	22
Corr. for G. M. T.		+2
L. S. T.	4	44
H. A.	4	40
R. A.	0	04

The right ascension and declination of the unknown star, as we have now approximately found them, are $0^h\ 04^m$ and $28°\ 49'$ N. The star is, therefore, Alpheratz (α Androm.), whose tabulated coördinates are right ascension $0^h\ 04^m\ 11^s$ and declination $28°\ 38'\ 36''$ N.

(The H. O. issues publication No. 127 Star Identification Tables, which enables one to find, by inspection, the name of all navigational stars, knowing the altitude and bearing by compass.)

CHAPTER XVI

THE TIDES

294. The disturbing forces caused by the attraction of the moon and the sun, acting upon the waters of the earth, produce the tides. These consist of the regular rise and fall of the water of the ocean. The average period between two successive transits of the moon across the same meridian is $24^h\ 50^m\ 28\frac{1}{3}^s$ which is the average interval between successive high waters.

295. *Definitions.* — When the water is rising it is *flood* tide, as it is the movement of a tidal current towards the shore or up a tidal stream; when falling it is *ebb* tide or the movement of a tidal current away from shore or down a tidal stream. It is high water at the time when the rising tide is highest and low water when the falling tide is lowest. *Spring* tides, which occur near the times of new and full moon, are the highest tides of the month. *Neap* tides are those that occur when the moon is in quadrature. At spring tides the interval between corresponding tides of successive days falls below the average time, being but $24^h\ 38^m$, hence the acceleration in the time of occurrence of high and low waters, which is called the *priming* of the tide. At neap tides the interval is about $25^h\ 6^m$, which is greater than the mean, causing a retardation in the time of occurrence of high and low waters, called the *lagging* of the tide. High and low water occur, on the average of the twenty-eight days comprising a lunar month, at about the same intervals after the transit of the moon over the meridian. These nearly constant intervals, expressed in hours and minutes, are known as the *high water lunitidal interval* and *low water lunitidal interval.*

The interval between the moon's meridian passage at any place and the time of the next succeeding high water, as observed on the days when the moon is at full or change, is called the *vulgar* (or *common*) *establishment* of that place, or, sometimes, just the *establishment.* This interval is frequently spoken of as the *time of high water on full and change days* (abbreviated

"H. W. F. & C."); for since, on such days, the moon's two transits (upper and lower) over the meridian occur about midnight and noon, the vulgar establishment then corresponds closely with the local times of high water. When more extended observations have been made, the average of all high water lunitidal intervals for at least a lunar month is taken to obtain what is called, in distinction to the vulgar establishment, the *corrected establishment* of the port, or *mean high water lunitidal interval.*

Having found the time of high water by applying the establishment to the time of moon's transit, the navigator may obtain the time of low water with a fair degree of approximation by adding or subtracting $6^h\ 13^m$ or one-fourth of a mean lunar day. The *range* of the tide is the difference in height between low water and high water. This term is often applied to the difference existing under average conditions, and may in such a case be designated as the *mean range* or *mean rise and fall* to distinguish it from the *spring range* or *neap range*, which are the ranges at spring and neap tides.

The age of the tide. — The greatest effect of the sun and moon on a tide is at the new or full moon, and the amount of time that elapses from the instant of new or full moon to the highest subsequent tide at a place is called the age of the tide.

296. *General tidal laws.* — Two high tides will occur every day at a given port or place.

When the moon is on the equator the two tides occurring every day will be nearly the same in height. The equatorial tides will be larger than those in high latitudes.

When the moon has a north or south declination the two tides will be unequal in height, the greatest difference occurring soon after the extreme declination of the moon to the northward or southward is reached. If a higher high water comes after the upper transit of the moon by a certain amount of time when extreme north declination of the moon is reached it will follow the lower transit by this same amount of time when the moon's declination is south. The above applies the same to the low waters.

The time of high water comes after the upper transit of the moon a certain number of hours which is equal to the establishment of the port. The time of the next low water $6^h\ 13^m$ after the high water, and the time of the following high water comes at a mean interval of $12^h\ 25^m$ after the first high water occurred.

297. *Time of high and low water.* — The best and most accurate way to find the time of high or low water at a port is to use the General Tide Tables of the World prepared and issued by the U. S. Coast Survey. These tables are as necessary and essential to the equipment of a good navigator as other navigational books, but the following examples will explain how an approximate time of high or low water can be obtained without the tables.

EXAMPLE 1: April 15, 1918, find the times of the low water just before noon and the high water just after noon at Portland, Maine, Lat. 43° 09′ N., Lon. 70° 15′ W.

	d	*h*	*m*
G. M. T. of Gr. upper transit	14	3	09
G. M. T. of Gr. upper transit	15	4	07
	2)29	7	16
G. M. T. of Gr. lower transit	14	15	38
Corr. for +70° Lon. (Tab. II) +			11
L. M. T. of local lower transit	14	15	49

	d	*h*	*m*
Transit (lower),	14	15	49
H. W. Lun. Int. (App. IV Bowd.)		11	06
	15	2	55
L. M. T., H. W.,	Apr. 15, 2.55 P.M.		

	d	*h*	*m*
Transit (lower)	14	15	49
L. W. Lun. Int. (App. IV)		4	51
	14	20	40
L. M. T., L. W.	Apr. 15, 8.40 A.M.		

From Coast Survey Tide Tables low water, April 15, 1918, 8.06 A.M. and high water 2.19 P.M.

EXAMPLE 2: February 17, 1918, find the time of high water at the Presidio, San Francisco, Cal., Lat. 37° 47′ N., 122° 27′ W. in the evening.

	d	*h*	*m*
G. M. T. of Gr. upper transit	16	4	42
G. M. T. of Gr. upper transit	17	5	38
	2)33	10	20
G. M. T. of Gr. lower transit	16	17	10
Corr. + 122° Lon. (Tab. 11)	+		19
L. M. T. local lower transit	16	17	29
H. W. Lun. Int. (App. IV)		11	43
L. M. T., H. W.	17	5	12
	Feb. 17, 5: 12 P.M.		

From Tide Tables high water February 17, 1918, 5.53 P.M.

EXAMPLE 3: May 21, 1918, find the time of low water at Singapore, Lat. 1° 18′ N.; Long. 103° 51′ E., in the afternoon.

	d	*h*	*m*
G. M. T. of Gr. upper transit	20	8	08
G. M. T. of Gr. upper transit	21	8	50
	2)41	16	58
G. M. T. of Gr. lower transit	20	20	29
Corr. for −104° Lon. (Tab. 11)	−		12
L. M. T. of local lower transit	20	20	17
L. W. Lun. Int. (App. IV)	+	4	02
L. M. T., L. W.	21	0	19
	May 21, 12: 19 P.M.		

From Tide Tables low water 1.24 P.M.

APPENDIX I

EXTRACTS FROM THE AMERICAN NAUTICAL ALMANAC (1918) WHICH HAVE REFERENCE TO THE EXAMPLES FOR THAT YEAR GIVEN IN THIS WORK.

Day of Month	Right Ascension of the Mean Sun at Greenwich Mean Noon																							
	January			February			March			April			May			June			July			September			November			December		
	h	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*	*h*	*m*	*s*
1	18	41	18.4	20	43	31.7	.....			.....			2	34	25.0														
8	.....			.....			.....			1	3	44.2																	
9	.....			.....			.....			.....			3	5	57.4														
10	.....			.....			.....			.....			.....			5	12	7.2											
13	.....			.....			.....			.....			.....			.....			7	22	13.6								
14	19	32	33.7																										
15	19	36	30.2	.....			.....			.....			.....			.....			.....			11	34	33.1						
16	.....			.....			.....			.....			.....			.....			7	34	3.3	.....			.....			17	37	16.2
17	.....			.....			.....			.....			.....			.....			.....			.....			15	42	56.0			
19	.....			.....			.....			.....			.....			.....			.....			11	50	19.3						
20	.....			.....			.....			.....			.....			.....			.....			11	54	15.9						
25	.....			.....			.....			.....			4	9	2.3														
26	20	19	52.4	22	22	5.6																								
27	20	23	48.9	22	26	2.1																								
30	20	35	38.6	22	37	51.8	0	28	15.2																					

NOTE. — For the *correction to be added to* R. A. M. S. at G. M. N. *for time past noon* or the reduction take from Table 8, and 9, Bowditch, which is the same as Table II, and Table III, Nautical Almanac.

SUN, 1918. Mean time of Sidereal Noon at Greenwich

January	28.....	3^h	31^m	39^s.8
"	29.....	3	27	43 .9
"	30.....	3	23	48 .0

SUN, JANUARY, 1918

G. M. T.	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time
	Thursday 3		Thursday 24		Monday 28			
h	° ′	*m* *s*	° ′	*m* *s*	° ′	*m* *s*		
0	−22 52.7	−4 22.5	−19 19.9	−12 7.3	−18 20.0	−13 0.1		
2	22 52.3	4 24.9	19 18.7	12 8.6	18 18.7	13 1.1		
4	22 51.8	4 27.2	19 17.5	12 9.8	18 17.4	13 2.0	SEMIDIAMETER	
6	22 51.3	4 29.5	19 16.3	12 11.0	18 16.1	13 3.0		
8	22 50.8	4 31.8	19 15.1	12 12.2	18 14.8	13 3.9		′
10	22 50.3	4 34.1	19 13.9	12 13.4	18 13.5	13 4.9	Jan. 1	16.30
12	22 49.9	4 36.4	19 12.7	12 14.6	18 12.1	13 5.8	11	16.29
14	22 49.4	4 38.7	19 11.5	12 15.8	18 10.8	13 6.7	21	16.28
16	22 48.9	4 41.0	19 10.3	12 17.0	18 9.5	13 7.7	31	16.26
18	22 48.4	4 43.3	19 9.1	12 18.2	18 8.2	13 8.6		
20	22 47.9	4 45.6	19 7.9	12 19.4	18 6.9	13 9.5		
22	22 47.4	4 47.9	−19 6.6	−12 20.6	−18 5.5	−13 10.4		
H. D.	0.2	1.2	0.6	0.6	0.7	0.5		

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.

SUN, MARCH, 1918

G. M. T.	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time
	Monday 4		Wednesday 20		Thursday 21			
h	° ′	*m* *s*	° ′	*m* *s*	° ′	*m* *s*		
0	−6 38.4	−11 59.0	−0 22.2	−7 46.0	+0 1.5	−7 28.0		
2	6 36.5	11 57.9	0 20.2	7 44.5	0 3.5	7 26.5	SEMIDIAMETER	
4	6 34.6	11 56.8	0 18.2	7 43.0	0 5.5	7 25.0		
6	6 32.7	11 55.7	0 16.2	7 41.5	0 7.5	7 23.5		
8	6 30.7	11 54.6	0 14.3	7 40.0	0 9.4	7 22.0		′
10	6 28.8	11 53.5	0 12.3	7 38.5	0 11.4	7 20.5	Mar. 1	16.17
12	6 26.9	11 52.4	0 10.3	7 37.0	0 13.4	7 19.0	11	16.13
14	6 25.0	11 51.3	0 8.3	7 35.5	0 15.4	7 17.5	21	16.08
16	6 23.0	11 50.2	0 6.4	7 34.0	0 17.3	7 16.0	31	16.04
18	6 21.1	11 49.1	0 4.4	7 32.5	0 19.3	7 14.4		
20	6 19.2	11 48.0	0 2.4	7 31.0	0 21.3	7 12.9		
22	−6 17.3	−11 46.8	−0 0.4	−7 29.5	0 23.3	7 11.4		
H. D.	1.0	0.6	1.0	0.7	1.0	0.8		

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.

SUN, MAY, 1918

G. M. T.	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time
	Tuesday 14		Wednesday 15		Monday 20	
h	° ′	*m* *s*	° ′	*m* *s*	° ′	*m* *s*
0	+18 29.8	+3 48.5	+18 44.3	+3 48.6	+19 52.0	+3 40.8
2	18 31.0	3 48.5	18 45.5	3 48.5	19 53.1	3 40.6
4	18 32.3	3 48.5	18 46.7	3 48.5	19 54.1	3 40.3
6	18 33.5	3 48.5	18 47.9	3 48.5	19 55.2	3 40.1
8	18 34.7	3 48.6	18 49.1	3 48.5	19 56.2	3 39.8
10	18 35.9	3 48.6	18 50.3	3 48.4	19 57.3	3 39.6
12	18 37.1	3 48.6	18 51.5	3 48.4	19 58.3	3 39.3
14	18 38.3	3 48.6	18 52.6	3 48.4	19 59.3	3 39.1
16	18 39.5	3 48.6	18 53.8	3 48.3	20 0.4	3 38.8
18	18 40.7	3 48.6	18 55.0	3 48.3	20 1.4	3 38.5
20	18 41.9	3 48.6	18 56.2	3 48.2	20 2.5	3 38.2
22	18 43.1	3 48.6	18 57.3	3 48.2	+20 3.5	+3 38.0
H. D.	0.6	0.0	0.6	0.0	0.5	0.1

Sun's Declination	Equation of Time
SEMIDIAMETER	
	′
May 1	15.90
11	15.86
21	15.83
31	15.80

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.

SUN, JUNE, 1918

G. M. T.	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time
	Sunday 30			
h	° ′	*m* *s*		
0	+23 13.0	−3 17.0		
2	23 12.7	3 18.0		
4	23 12.4	3 19.0	SEMIDIAMETER	
6	23 12.1	3 20.0		
8	23 11.8	3 21.0		′
10	23 11.5	3 22.0	June 1	15.80
12	23 11.2	3 23.0	11	15.78
14	23 10.9	3 24.0	21	15.77
16	23 10.6	3 24.9	July 1	15.76
18	23 10.3	3 25.9		
20	23 10.0	3 26.9		
22	+23 9.7	−3 27.9		
H. D.	0.2	0.5		

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.

SUN, JULY, 1918

G. M. T.	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time
	Wednesday 3		Saturday 13		Tuesday 16		Friday 19	
h	° ′	*m* *s*	° ′	*m* *s*	° ′	*m* *s*	° ′	*m* *s*
0	+23 1.0	−3 52.0	+21 55.2	−5 27.7	+21 28.0	−5 48.0	+20 57.5	−6 3.5
2	23 0.6	3 52.9	21 54.5	5 28.4	21 27.2	5 48.5	20 56.6	6 3.9
4	23 0.2	3 53.8	21 53.8	5 29.0	21 26.4	5 49.0	20 55.7	6 4.3
6	22 59.8	3 54.8	21 53.0	5 29.6	21 25.6	5 49.5	20 54.8	6 4.6
8	22 59.4	3 55.7	21 52.3	5 30.2	21 24.8	5 50.0	20 53.9	6 5.0
10	22 59.0	3 56.6	21 51.6	5 30.8	21 24.0	5 50.4	20 53.0	6 5.3
12	22 58.6	3 57.6	21 50.9	5 31.4	21 23.1	5 50.9	20 52.1	6 5.6
14	22 58.2	3 58.5	21 50.2	5 32.0	21 22.3	5 51.4	20 51.2	6 6.0
16	22 57.8	3 59.4	21 49.4	5 32.6	21 21.5	5 51.9	20 50.3	6 6.3
18	22 57.4	4 0.4	21 48.7	5 33.2	21 20.7	5 52.3	20 49.4	6 6.6
20	22 57.0	4 1.3	21 48.0	5 33.8	21 19.9	5 52.8	20 48.5	6 7.0
22	22 56.6	4 2.2	21 47.2	5 34.4	+21 19.0	−5 53.2	20 47.6	6 7.3
H. D.	0.2	0.5	0.4	0.3	0.4	0.2	0.5	0.2

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.
S. D. July 1, 15′.76, July 11, 15′.76, July 21, 15′.77, July 31, 15′.79

SUN, AUGUST, 1918

G. M. T.	Sun's Declination	Equation of Time
	Tuesday 20	
h	° ′	*m* *s*
0	+12 38.3	−3 26.5
2	12 36.7	3 25.3
4	12 35.0	3 24.1
6	12 33.4	3 23.0
8	12 31.8	3 21.8
10	12 30.1	3 20.6
12	12 28.5	3 19.4
14	12 26.8	3 18.2
16	12 25.2	3 17.0
18	12 23.5	3 15.9
20	12 21.9	3 14.7
22	+12 20.2	−3 13.5
H. D.	0.8	0.6

SEMIDIAMETER	
	′
Aug. 1	15.79
11	15.81
21	15.84
31	15.88

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.

SUN, SEPTEMBER, 1918

G. M. T.	Sun's Declination	Equation of Time	Sun's Declination	Equation of Time
	Tuesday 3		Friday 20	
h	° ′	*m* *s*	° ′	*m* *s*
0	+7 45.6	+0 29.2	+1 18.6	+6 20.7
2	7 43.8	0 30.8	1 16.6	6 22.5
4	7 42.0	0 32.4	1 14.7	6 24.3
6	7 40.1	0 34.0	1 12.7	6 26.0
8	7 38.3	0 35.6	1 10.8	6 27.8
10	7 36.4	0 37.2	1 8.8	6 29.6
12	7 34.6	0 38.8	1 6.9	6 31.4
14	7 32.8	0 40.5	1 5.0	6 33.1
16	7 30.9	0 42.1	1 3.0	6 34.9
18	7 29.1	0 43.7	1 1.1	6 36.7
20	7 27.3	0 45.3	0 59.1	6 38.4
22	7 25.4	0 46.9	+0 57.2	+6 40.2
H. D.	0.9	0.8	1.0	0.9

SEMIDIAMETER	
	′
Sept. 1	15.88
11	15.92
21	15.97
Oct. 1	16.01

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.

SUN, NOVEMBER, 1918

G. M. T.	Sun's Declination	Equation of Time
	Wednesday 20	
h	° ′	*m* *s*
0	−19 34.7	+14 25.9
2	19 35.9	14 24.8
4	19 37.0	14 23.6
6	19 38.2	14 22.5
8	19 39.3	14 21.3
10	19 40.4	14 20.1
12	19 41.6	14 18.9
14	19 42.7	14 17.7
16	19 43.9	14 16.6
18	19 45.0	14 15.4
20	19 46.1	14 14.1
22	−19 47.2	+14 12.9
H. D.	0.6	0.6

SEMIDIAMETER	
	′
Nov. 1	16.15
11	16.19
21	16.23
Dec. 1	16.25

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.

SUN, DECEMBER, 1918

G. M. T.	Sun's Declination	Equation of Time
	Tuesday 3	
h	° ′	*m* *s*
0	−22 2.1	+10 20.3
2	22 2.8	10 18.3
4	22 3.5	10 16.3
6	22 4.2	10 14.4
8	22 4.9	10 12.4
10	22 5.7	10 10.4
12	22 6.4	10 8.4
14	22 7.1	10 6.4
16	22 7.8	10 4.5
18	22 8.5	10 2.5
20	22 9.2	10 0.5
22	22 9.9	9 58.5
H. D.	0.4	1.0

SEMIDIAMETER	
	′
Dec. 1	16.25
11	16.28
21	16.29
31	16.30

NOTE. — The Equation of Time is to be applied to the G. M. T. in accordance with the sign as given.

G. M. T.	Right Ascension		Declination		S. D.	H. P.
	January 19					
h	h m s		° ′		′	′
0	1 34 0	273	+14 52.2	236	16.0	58.8
2	1 38 32	273	15 14.8	232	16.0	58.7
4	1 43 5	273	15 37.1	210	16.0	58.7
6	1 47 38	274	15 59.0	216	16.0	58.6
8	1 52 12	274	16 20.5	212	16.0	58.6
10	1 56 46	275	16 41.7	207	16.0	58.5
12	2 1 21	276	17 2.4	202	16.0	58.5
14	2 5 56	276	17 22.6	199	15.9	58.4
16	2 10 32	277	17 42.5	194	15.9	58.4
18	2 15 9	277	18 1.9	190	15.9	58.3
20	2 19 46	278	18 20.9	185	15.9	58.3
22	2 24 24	278	18 39.4	181	15.9	58.2
	January 27					
0	8 49 30	287	+14 19.8	208	15.0	54.8
2	8 53 27	286	13 59.3	208	14.9	54.8
4	8 57 23	285	13 38.5	210	14.9	54.7
9	9 1 18	285	13 17.5	211	14.9	54.7
8	9 5 13	283	12 56.4	214	14.9	54.7
10	9 9 6	283	12 35.0	216	14.9	54.6
12	9 12 58	282	12 13.4	217	14.9	54.6
14	9 16 50	280	11 51.7	219	14.9	54.6
16	9 20 40	280	11 29.8	221	14.9	54.6
18	9 24 30	279	11 7.7	222	14.9	54.5
20	9 28 19	278	10 45.5	224	14.9	54.5
22	9 32 7	278	10 23.1	224	14.9	54.5

G. M. T.	Right Ascension		Declination		S. D.	H. P.
	January 25					
h	h m s		° ′		′	′
0	7 9 25	303	+21 F8.7	121	15.2	55.6
2	7 13 47	303	20 55.6	125	15.2	55.5
4	7 18 9	303	20 42.1	138	15.2	55.5
6	7 22 30	300	20 28.3	143	15.1	55.5
8	7 26 50	298	20 14.0	146	15.1	55.4
10	7 31 8	296	19 59.4	151	15.1	55.4
12	7 35 26	294	19 44.3	155	15.1	55.4
14	7 39 42	293	19 29.0	158	15.1	55.3
16	7 43 57	294	19 13.2	160	15.1	55.3
18	7 48 11	288	18 57.2	164	15.1	55.3
20	7 52 24	288	18 40.8	168	15.1	55.2
22	7 56 36	290	18 24.0	171	15.1	55.2
	January 28					
0	9 35 55	226	+10 0.6	227	14.9	54.5
2	9 39 41	226	9 37.9	228	14.9	54.4
4	9 43 27	226	9 15.1	229	14.9	54.4
6	9 47 13	224	8 52.2	230	14.8	54.4
8	9 50 57	224	8 29.2	233	14.8	54.4
10	9 54 41	223	8 6.0	233	14.8	54.4
12	9 58 24	223	7 42.8	234	14.8	54.3
14	10 2 7	222	7 19.4	234	14.8	54.3
16	10 5 49	222	6 56.0	236	14.8	54.3
18	10 9 31	221	6 32.4	236	14.8	54.3
20	10 13 12	220	6 8.8	237	14.8	54.3
22	10 16 52	221	5 45.1	237	14.8	54.2
24	10 20 33		+5 21.4		14.8	54.2

G. M. T.	Right Ascension		Declination		S. D.	H. P.
	July 28					
h	h m s		° ′		′	′
0	0 18 45	270	+7 25.8	270	16.3	59.7
2	0 23 15	270	7 52.8	267	16.3	59.7
4	0 27 45	270	8 19.5	266	16.3	59.7
6	0 32 15	271	8 46.1	263	16.3	59.7
8	0 36 46	271	9 12.4	261	16.3	59.7
10	0 41 17	272	9 38.5	258	16.3	59.6
12	0 45 49	272	10 4.3	256	16.3	59.6
14	0 50 21	273	10 29.9	253	16.3	59.6
16	0 54 54	273	10 55.2	251	16.3	59.6
18	0 59 27	274	11 20.3	247	16.3	59.5
20	1 4 1	274	11 45.0	245	16.2	59.5
22	1 8 35	276	12 9.5	243	16.2	59.5
	September 19					
0	22 42 3	277	−2 51.3	289	16.6	60.9
2	22 46 40	278	2 22.4	291	16.6	60.9
4	22 51 18	278	1 53.3	291	16.6	60.9
6	22 55 56	278	1 24.2	292	16.6	61.0
8	23 0 34	278	0 55.0	292	16.6	61.0
10	23 5 12	279	−0 25.8	288	16.7	61.0
12	23 9 51	279	+0 3.5	288	16.7	61.1
14	23 14 30	279	0 32.8	288	16.7	61.1
16	23 19 9	280	1 2.1	284	16.7	61.1
18	23 23 49	280	1 31.5	286	16.7	61.2
20	23 28 29	280	2 0.8	284	16.7	61.2
22	23 33 9	281	2 30.1	292	16.7	61.2

First Quarter, Jan. 19*d* 2*h* 38*m*.

Full Moon, Jan. 26*d* 15*h* 14*m*.

MOON, 1918

TIME OF TRANSIT, MERIDIAN OF GREENWICH

Date	Greenwich Mean Time	Date	Greenwich Mean Time	Date	Greenwich Mean Time	Date	Greenwich Mean Time
	h m		*h m*		*h m*		*h m*
Jan. 20	6 48 54	Feb. 16	4 42 56	May 19	7 27 41	July 27	15 40 52
21	7 42 56	17	5 38 56	20	8 8 41	28	16 32 58
22	8 38 54	18	6 34 54 60	21	8 50 43 43	29	17 25 55
24	10 26 50	Apr. 14	3 9 58	22	9 33 46	30	18 20 57
25	11 16 48	15	3 9 55	23	10 19 48		
26	12 4 45	16	5 2 51				
27	12 49 42						
28	13 31 41						
29	14 12 41						
30	14 53 41						

VENUS, 1918

GREENWICH MEAN TIME

Date	Apparent Right Ascension (Noon)		Apparent Declination (Noon)		Transit, Meridian of Greenwich
	h m s		° ′		h m
Jan. 4	21 44 20	114	13 2.0	205	2 51
5	21 46 6	106	12 41.7	203	2 49
		100		200	
6	21 47 46	92	−12 21.7	199	2 46
14	21 56 33	37	9 51.9	174	2 24
15	21 57 3	30	9 35.1	168	2 20
		20		164	
16	21 57 23	11	− 9 18.7	157	2 17
17	21 57 34	2	9 3.0	151	2 13

Semidiameter: Jan. 1, 0′.33; Feb. 1, 0′.51; Hor. Parallax: Jan. 1, 0′.34; Feb. 1, 0′.52.

JUPITER, 1918

GREENWICH MEAN TIME

Date	Apparent Right Ascension (Noon)		Apparent Declination (Noon)		Transit, Meridian of Greenwich
	h m s		° ′		h m
Jan. 1	4 2 29	21	+19 54.3	7	9 20
2	4 2 8	19	19 53.6	7	9 15
15	3 58 54	10	19 47.5	3	8 21
		9		1	
16	3 58 45	9	+19 47.4	2	8 17
20	3 58 16	6	19 47.0	0	8 1
		5		0	
21	3 58 11	4	+19 47.0	1	7 57
22	3 58 7	3	19 47.1	1	7 53
23	3 58 4	3	19 47.2	1	7 49
24	3 58 1	1	19 47.3	2	7 45
25	3 58 0	1	19 47.5	2	7 41
26	3 57 59	0	+19 47.7	3	7 37
27	3 57 59	2	19 48.0	3	7 33
28	3 58 1	1	19 48.3	3	7 29
29	3 58 2	3	19 48.6	4	7 25
Dec. 16	6 56 19	31	+22 48.7	8	13 17
17	6 55 48	32	22 49.5	9	13 12

Polar Semidiameter: Jan. 1, 0′.37; Feb. 1, 0′.34; Hor. Parallax: Jan. 1, 0′.03; Feb. 1, 0′.03.

APPARENT PLACES OF STARS, 1918

FOR THE UPPER TRANSIT AT GREENWICH

No.	Constellation Name	Right Ascension													
				Jan. 1	Feb. 1	Mar. 1	Apr. 1	May 1	June 1	July 1	Aug. 1	Sept. 1	Oct. 1	Nov. 1	Dec. 1	Dec. 32
		h	*m*	*s*	*s*	*s*	*s*	*s*	*s*	*s*	*s*	*s*	*s*	*s*	*s*	*s*
1	α Androm.	0	4	9.6	9.2	9.0	9.1	9.7	10.6	11.6	12.6	13.2	13.5	13.4	13.1	12.8
5	α Urs. Min.	1	29	129.0	96.1	70.6	56.8	62.2	84.1	115.8	151.5	181.7	200.4	205.1	194.2	168.5
12	α Aurigæ	5	10	41.1	40.9	40.3	39.6	39.1	39.1	39.7	40.7	41.9	43.2	44.3	45.1	45.5
17	α Can. Maj.	6	41	34.3	34.4	34.0	33.5	32.9	32.7	32.8	33.3	34.0	34.8	35.7	36.4	36.9
18	ϵ Can. Maj.	6	55	26.5	26.6	26.2	25.6	25.0	24.6	24.6	25.0	25.7	26.6	27.6	28.4	28.9
19	α Can. Min.	7	35	3.0	3.3	3.1	2.7	2.2	1.9	1.9	2.3	2.9	3.7	4.6	5.5	6.1
20	β Gemin.	7	40	20.8	21.2	21.0	20.5	20.0	19.7	19.6	20.1	20.7	21.6	22.6	23.6	24.4
25	α Leonis	10	4	2.4	3.1	3.4	3.3	2.9	2.5	2.3	2.2	2.4	2.8	3.6	4.6	5.5
33	α Virginis	13	20	53.1	54.2	54.9	55.3	55.5	55.5	55.2	54.9	54.6	54.4	54.7	55.3	56.3
34	θ Centauri	14	1	51.7	52.9	53.9	54.5	54.9	55.0	54.7	54.2	53.8	53.5	53.6	54.2	55.3
35	α Boötis	14	11	55.7	56.7	57.5	58.1	58.4	58.4	58.2	57.7	57.3	57.0	57.0	57.4	58.3
40	α Scorpii	16	24	22.7	23.7	24.6	25.5	26.3	26.8	27.0	26.8	26.3	25.8	25.5	25.7	26.3
47	α Lyræ	18	34	8.7	9.2	10.0	11.0	12.0	12.7	13.2	13.2	12.6	11.9	11.2	10.7	10.7

APPARENT PLACES OF STARS, 1918

FOR THE UPPER TRANSIT AT GREENWICH

No.	Declination													Special Name	Mag.
		Jan. 1	Feb. 1	Mar. 1	Apr. 1	May 1	June 1	July 1	Aug. 1	Sept. 1	Oct. 1	Nov. 1	Dec. 1	Dec. 32	
	°	′	′	′	′	′	′	′	′	′	′	′	′	′	
1	+28	38.5	38.5	38.4	38.3	38.3	38.3	38.4	38.5	38.7	38.8	38.8	38.9	38.8	Alpheratz	2.2
5	+88	52.5	52.5	52.4	52.3	52.1	52.0	52.0	52.0	52.1	52.3	52.5	52.6	52.8	Polaris	2.1
12	+45	55.1	55.1	55.1	55.1	55.1	55.0	54.9	54.9	54.9	54.9	54.9	55.0	55.1	Capella	0.2
17	−16	36.2	36.4	36.4	36.4	36.4	36.4	36.3	36.2	36.1	36.1	36.1	36.2	36.4	Sirius	−1.6
18	−28	51.6	51.8	51.9	51.9	51.9	51.8	51.7	51.6	51.5	51.4	51.5	51.6	51.8	Adhara	1.6
19	+5	26.1	26.0	26.0	26.0	26.0	26.0	26.0	26.1	26.1	26.1	26.0	26.0	25.9	Procyon	0.5
20	+28	13.4	13.4	13.4	13.5	13.5	13.5	13.4	13.4	13.4	13.3	13.3	13.2	13.2	Pollux	1.2
25	+12	21.9	21.9	21.8	21.9	21.9	21.9	21.9	21.9	21.9	21.9	21.8	21.7	21.6	Regulus	1.3
33	−10	44.1	44.2	44.3	44.3	44.3	44.3	44.3	44.3	44.3	44.2	44.2	44.3	44.4	Spica	1.2
34	−35	58.0	58.0	58.1	58.3	58.3	58.4	58.4	58.4	58.4	58.3	58.2	58.2	58.2		2.3
35	+19	36.3	36.2	36.2	36.2	36.3	36.3	36.4	36.4	36.4	36.4	36.3	36.2	36.0	Arcturus	0.2
40	−26	15.1	15.1	15.1	15.2	15.2	15.2	15.2	15.2	15.2	15.2	15.2	15.1	15.1	Antares	1.2
47	+38	42.4	42.2	42.1	42.1	42.2	42.3	42.5	42.6	42.7	42.7	42.7	42.7	42.5	Vega	0.1

MERIDIAN TRANSIT OF STARS, 1918

GREENWICH MEAN TIME OF TRANSIT AT GREENWICH

Constellation Name	Mag.	Jan. 1	Feb. 1	Mar. 1	Apr. 1	May 1	June 1	July 1	Aug. 1	Sept. 1	Oct. 1	Nov. 1	Dec. 1
		h m	*h m*	*h m*	*h m*	*h m*	*h m*	*h m*	*h m*	*h m*	*h m*	*h m*	*h m*
α Androm.	2.2	5 22	3 20	1 30	23 24	21 26	19 24	17 26	15 25	13 23	11 25	9 23	7 25
α Urs. Min.	2.1	6 49	4 46	2 56	0 54	22 52	20 50	18 53	16 52	14 50	12 52	10 51	8 53
α Aurigæ	0.2	10 28	8 26	6 36	4 34	2 36	0 34	22 32	20 30	18 28	16 30	14 29	12 31
α Can. Maj.	−1.6	11 58	9 56	8 6	6 4	4 6	2 5	0 7	22 1	19 59	18 1	15 59	14 1
α Can. Min.	0.5	12 52	10 50	9 0	6 58	5 0	2 58	1 0	22 54	20 52	18 54	16 53	14 55
α Leonis	1.3	15 20	13 18	11 28	9 26	7 28	5 27	3 29	1 27	23 21	21 23	19 21	17 23
α Virginis	1.2	18 37	16 35	14 45	12 43	10 45	8 43	6 45	4 43	2 41	0 43	22 37	20 39
θ Centauri	2.3	19 17	17 16	15 25	13 24	11 26	9 24	7 26	5 24	3 22	1 24	23 18	21 20
α Boötis	0.2	19 27	17 26	15 36	13 34	11 36	9 34	7 36	5 34	3 32	1 34	23 28	21 30
α Scorpii	1.2	21 40	19 38	17 48	15 46	13 48	11 46	9 48	7 46	5 44	3 46	1 44	23 42
α Lyræ	0.1	23 49	21 47	19 57	17 55	15 57	13 55	11 57	9 56	7 54	5 56	3 54	1 56

Extracts from Ephemeris

SUN, 1918

FOR GREENWICH MEAN NOON

Date	Day of the Week	Apparent Right Ascension	Var. per Hour	Apparent Declination	Var. per Hour	Semi-diameter	Hor. Par	Equation of Time App. — Mean	Var. per Hour	Sidereal Time or Right Ascension of Mean Sun
		h m s	*s*	° ′ ″	″	′ ″	″	*m s*	*s*	*h m s*
Jan. 10	Th.	19 24 14.40	10.884	−22 2 17.2	+21.87	16 17.68	8.95	−7 26.97	−1.027	19 16 47.44
11	Fr.	19 28 35.34	10.860	21 53 19.4	22.94	16 17.63	8.95	7 51.35	1.004	19 20 43.99
30	We.	20 49 0.24	10.272	17 48 3.8	40.74	16 15.95	8.93	13 21.67	0.415	20 35 38.58
31	Th.	20 53 6.36	10.238	17 31 36.6	41.52	16 15.81	8.93	13 31.23	−0.381	20 39 35.13
Feb. 3	Su.	21 5 19.88	10.137	−16 40 24.5	+43.79	16 15.36	8.93	−13 55.08	−0.281	20 51 24.80
4	Mo.	21 9 22.78	10.104	16 22 44.9	44.51	16 15.20	8.93	14 1.42	0.248	20 55 21.36
May 1	We.	2 31 29.69	9.531	+14 54 42.7	+45.71	15 54.06	8.73	+2 55.27	+0.325	2 34 24.96
30	Th.	4 25 58.72	10.185	21 41 28.9	22.82	15 48.36	8.68	2 46.36	0.329	4 28 45.09
31	Fr.	4 30 3.39	10.204	21 50 25.4	21.88	15 48.21	8.68	2 38.25	0.347	4 32 41.64
June 1	Sa.	4 34 8.51	10.222	21 58 59.1	20.93	15 48.07	8.68	2 29.69	0.366	4 36 38.20
July 2	Tu.	6 42 32.02	10.337	23 5 21.6	−10.51	15 45.71	8.66	3 40.54	0.481	6 38 51.48
3	We.	6 46 40.00	10.327	23 0 57.3	11.52	15 45.70	8.66	3 51.96	0.470	6 42 48.04

EXTRACTS FROM EPHEMERIS

SUN, 1918

FOR WASHINGTON APPARENT NOON

Date	Apparent Right Ascension	Var. per Hour	Apparent Declination	Var. per Hour	Equation of Time Mean-App.	Var. per Hour	Semi-diameter	S. T. of Sem Pass Merid.	Sidereal Time of Mean Noon
	h m s	*s*	° ′ ″	″	*m s*	*s*	′ ″	*m s*	*h m s*
Jan. 10	19 25 11.68	10.882	−22 0 21.5	+22.11	+7 32.36	+1.022	16 17.67	1 10.50	19 17 38.08
11	19 29 32.56	10.858	21 51 17.9	23.18	7 56.62	0.998	16 17.62	1 10.43	19 21 34.63
Apr. 14	1 28 37.41	9.223	9 17 26.1	54.12	0 23.18	0.631	15 58.30	1 4.90	1 28 14.18
15	1 32 18.93	9.237	9 39 0.4	53.72	0 8.17	0.618	15 58.04	1 4.95	1 32 10.74
May 30	4 26 50.59	10.190	+21 43 24.6	22.62	−2 44.68	0.333	15 48.33	1 8.24	4 29 35.73
31	4 30 55.38	10.209	21 52 16.3	21.68	−2 36.47	0.351	15 48.18	1 8.30	4 33 32.28
June 24	6 10 15.04	10.391	23 25 42.5	2.54	2 5.03	0.533	15 45.95	1 8.93	6 8 9.66
25	6 14 24.38	10.387	23 24 29.2	3.57	2 17.78	0.529	15 45.90	1 8.91	6 12 6.22

FORMS FOR WORKING DEAD RECKONING AND ASTRONOMICAL SIGHTS

Form for Day's Work, Dead Reckoning

Compass Course	Var.	Dev.	Lee-way	Error	True Course	Patent log	Dist.	N.	S.	E.	W.	Long

	Latitude ° ′ ″		*Longitude* ° ′ ″	
Left at departure (or noon)	.....	N. *or* S.	.....	E. *or* W.
Run to.....	.....	N. *or* S.	.....	E. *or* W.
By D. R. at.....	.....	N. *or* S.	.....	E. *or* W.
Run to.....	.....	N. *or* S.	.....	E. *or* W.
By D. R. at.....	.....	N. *or* S.	.....	E. *or* W.

Form for Time Sight of Sun's Lower Limb (Sumner Line) (chord method)

	h m s		° ′ ″		° ′ ″		° ′ ″		m s		m s
W. T.	.....	Obs. alt.	☉	Dec.	..... N. *or* S.	H. D.	±	Eq. t.	.....	H. D.	± .....
C–W	+	Corr.	±	Corr.	±	G. M. T.	.....	Corr.	±	G. M. T.	.....
Chro. t.	.....	*h*	.....	☽	.....	Corr.	±			Corr.	± .....
C. C.	±		° ′ ″								
G. M. T.	.....	Tab. 46 (Bowd.)	±								
Eq. t.	±	I. C.	±								
G. A. T.	.....	Corr.	±								

	° ′ ″			° ′ ″		
h	.....					
L_1	.....	sec	.....	L_2	.....	sec	.....
☽	.....	cosec	.....			cosec	.....
	2).....						
s_1	.....	cos	.....	s_2	.....	cos	.....
$s_1 - h$	.....	sin	.....	$s_2 - h$	.....	sin	.....
	h m s				*h m s*		
G. A. T.	.....			G. A. T.	.....		
$L.A.T._1$	.....	hav t_1	.....	$L.A.T._2$	.....	hav *h*	.....
Long.$_1$	{ *h m s* ..... ° ′ ″ ..... } E. *or* W.			Long.$_2$	{ *h m s* ..... ° ′ ″ ..... } E. *or* W.		

Form for Time Sight of a Star (Sumner Line) (Tangent Method)

		h m s
W. T.		.....
C–W	+	.....
Chro.		.....
C. C.	±	.....
G. M. T.		.....
R. A. M. S.	+	.....
Red. (Tab. 9)	+	.....
G. S. T.		.....
R. A. *		.....
H. A. from Gr.		..... E. *or* W.

		° ′ ″
Obs. alt. *		.....
Corr.	±	.....
h		.....
(Tab. 46 Bowd.)		′ ″
I. C.	±	.....
Corr.	±	.....
		° ′ ″
h		.....
L		.....
p		.....
		2).....
s		.....
s — *h*		.....
		h m s
Gr. H. A.		.....
H. A.		.....
Long.		{ ..... ° ′ ″ ..... } E. *or* W.

	h m s
R. A.	.....
	° ′ ″
Dec.	..... N. *or* S.
	° ′ ″
p	.....
	″ ′ ″
sec	.....
cosec	.....
cos	.....
sin	.....
E. *or* W.	———
E. *or* W. hav. *t*	.....

Azimuth from Tables

Form for Time Sight of a Planet (Sumner Line)

	h m s		° ′ ″		h m s		h m s		° ′ ″		° ′ ″
W. T.	.....	Obs. alt. *	.....	R. A.	.....	H. D.	±	Dec.	..... N. *or* S.	H. D.	± .....
C–W	+	Corr.	±	Corr.	±	G. M. T.	.....	Corr.	±	G. M. T.	.....
Chro. t	.....	*h*	.....			Corr.	±	*p*	.....	Corr.	± .....
C. C.	±		′ ″								
G. M. T.	.....	(Tab. 46)	.....								
R. A. M. S.	+	I. C.	±								
Red. (Tab. 9)	+										
		Corr.	±								
G. S. T.	.....										
R. A. *	.....										
H. A. from Gr.	.....	E. *or* W.									

For remainder of work, to find hour angle, longitude, and azimuth use method "Form for Time Sight of a Star (Sumner Line)."

FORMS FOR WORK

Form for Time Sight of Moon's Lower Limb (Sumner Line)

	h m s		° ′ ″		h m s				° ′ ″		° ′ ″
W. T.	.....	Obs. alt.	☾	R. A.	.....	H. D.	+	Dec.	.....N. *or* S.	H. D.	± .....
C — W	+	1st Corr.	±	Corr.	±	G. M. T.	±	Corr.	±	G. M. T.	± .....
Chro. t.	.....	Approx. alt.	.....								
C. C.	±	2d corr.	+			Corr.	±	*p*	.....	Corr.	± .....
G. M. T.	.....	*h*	.....								
R. A. M. S.	+	S. D.	±								
Red. (Tab. 9)	+	Aug.	+								
		I. C.	±								
		dip	−								
G. S. T.	.....										
R. A.	☾	1st corr.	±								
		p & *r* (Tab. 24)	+ ° ′ ″								
			.....								
H. A. from Gr.	.....										
E. *or* W.		2d corr.	+								

For remainder of work, to find hour angle, longitude, and azimuth use method "Form for Time Sight of Star (Sumner Line)."

Form for Meridian Altitude of Sun's Lower Limb

	° ′ ″		′ ″		h m s		° ′ ″		° ′ ″
Obs. alt. ☉	.....	Tab. 46	±	L. A. T.	.....	Dec.	..... N. *or* S.	H. D.	± .....
Corr.	±	I. C.	±	Long.	.....	Corr.	±	G. M. T.	± .....
h	.....	Corr.	±	G. A. T.	.....			Corr.	± .....
	° ′ ″			Eq. t.	.....				
s	..... N. *or* S.			G. M. T.	.....				
d	..... N. *or* S.								
Lat.	..... N. *or* S.								

Form for Meridian Altitude of a Star

	° ′ ″		′ ″		° ′ ″
Obs. alt. *	.....	I. C.	±	Dec.	..... N. *or* S.
Corr.	±	dip	−		
		ref.	−		
h	.....	Corr.	±		
s	..... N. *or* S.				
d	..... N. *or* S.				
Lat.	..... N. *or* S.				

Form for Meridian Altitude of a Planet

	° ′ ″		′ ″		h m		° ′ ″
Obs. alt. *	.....	Tab. 46 ±	.....	G. M. T., Gr. trans.	.....	Dec.	..... N. *or* S.
Corr. ±	.....	I. C. +	.....	Corr. for Long. ±	.....	Corr. ±	.....
h	.....	Corr. ±	.....	L. M. T., local trans.	.....	H. D. ±	.....
	° ′ ″			Long. ±	.....	G. M. T.	.....
z	..... N. *or* S.			G. M. T., local trans.	.....	Corr. ±	.....
d	..... N. *or* S.						
Lat.	..... N. *or* S.						

"Constant" Form for Meridian Altitude of a Body

	± 90° 00′ 00″
Dec.	± .....
Corr.	± .....
Constant	± .....
Obs. Alt.	± .....
Lat.	..... N. *or* S.

Rules for Signs

Case I.	Lat. & Dec. same name, Lat. greater	+90°	+ Dec. − Corr. − Alt.
Case II.	Lat. & Dec. same name, Dec. greater	−90°	+ Dec. + Corr. + Alt.
Case III.	Lat. and Dec. opposite names	+90°	− Dec. − Corr. − Alt.
Case IV.	Lower transit	+90°	− Dec. + Corr. + Alt.

Form for Latitude Sights of Sun's Lower Limb (Sumner Line)

	h m s		° ′ ″		° ′ ″		° ′ ″		*m s*		*m s*
W. T.	.....	Obs. alt. ⊙	.....	Dec.	..... N. *or* S.	H. D.	.....	Eq. t.	.....	H. D.	.....
C—W	±	Corr.	±	Corr.	±		*h.*	Corr.	±		
Chro. t.	.....	*h*	.....			G. M. T.	.....			G. M. T.	.....
C. C.	±		′ ″			Corr.	±			Corr.	± .....
G. M. T.	.....	Tab. 46	±								
Eq. t.	±	I. C.	+								
G. A. T.	.....	Corr.	±								
Long.	±										
L. A. T.	.....										
t	*h m s* ..... ° ′ ″										

φ′ φ″ Method

	° ′ ″			
h	.....	sec	.....	
d	.....	tan	.....	cosec .....
h	.....			sin .....
φ″	..... N. *or* S.	tan	.....	sin .....
φ′	..... N. *or* S.			cos .....
Lat.	..... N. *or* S.			

(Azimuth from Tables)

Reduction to Meridian

	″	
a	.....	
	° ′ ″	
h	.....	
at²	±	
H	.....	
	° ′ ″	
s	..... N. *or* S.	
d	..... N. *or* S.	
Lat.	..... N. *or* S.	

Form for Finding the Zenith Distance and the Intercept for Laying Down the Sumner Line by the Method of Saint Hilaire from a Sight of the Sun's Lower Limb

(COSINE — HAVERSINE FORMULA)

	h m s		° ′ ″		° ′ ″		*m s*		*m s*
W. T.	.....	Dec.	..... N. *or* S.	H. D.	.....	Eq. t.	.....	H. D.	.....
C—W +	.....	Corr.	±	G. M. T.	.....	Corr.	±	G. M. T.	.....
Chro. t.	.....			Corr.	±				*s*
C. C.	±							Corr.	± .....
G. M. T.	.....								
Eq. t.	±								
G. A. T.	.....								
Long. of assumed Pos.	..... E. *or* W.								
	h m s						° ′ ″		
L. A. T. = *t*	.....	hav	.....			Obs. alt. ☉	.....		
	° ′ ″					I. C.	+		
L	.....	cos	.....			Corr. (Tab. 46)	±		
d	.....	cos	.....			*h*	.....		
		hav θ				*s*	.....		
		nat hav θ	.....						
L − *d*	.....	nat hav	.....						
Comp. *s*	.....	nat hav	.....						
true *s*	.....								
Intercept	.....								

Form for Finding the Time of High (or Low) Water

	d h m
G. M. T. of Greenwich transit	.....
Corr. for Long. (Tab. 11)	± .....
L. M. T. of local transit	.....
Lunitidal int. (App. IV Bowd.)*	+ .....
L. M. T. of high (or low) water	.....

* or from chart.

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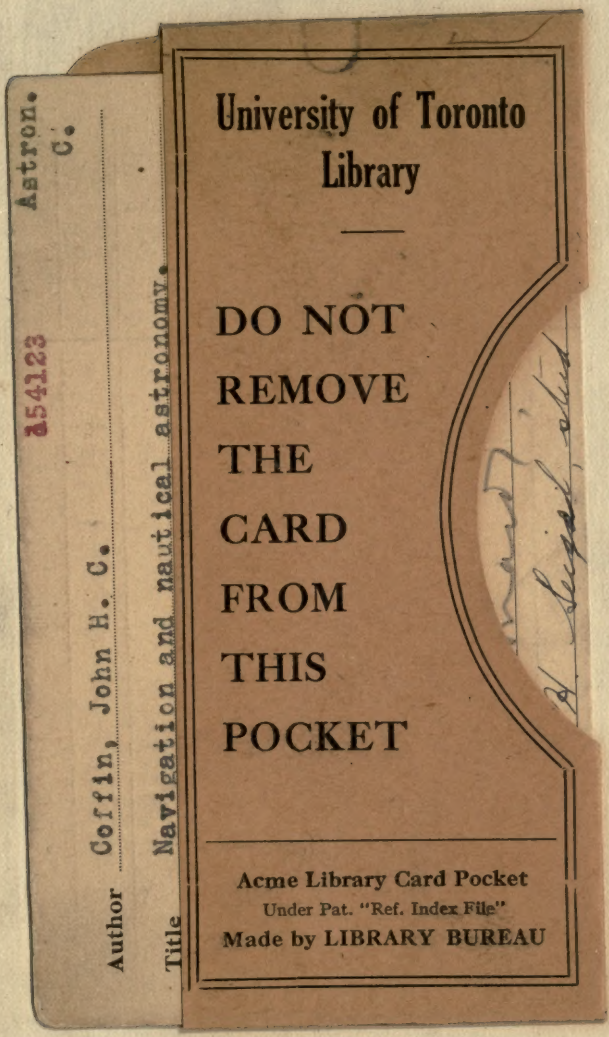
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